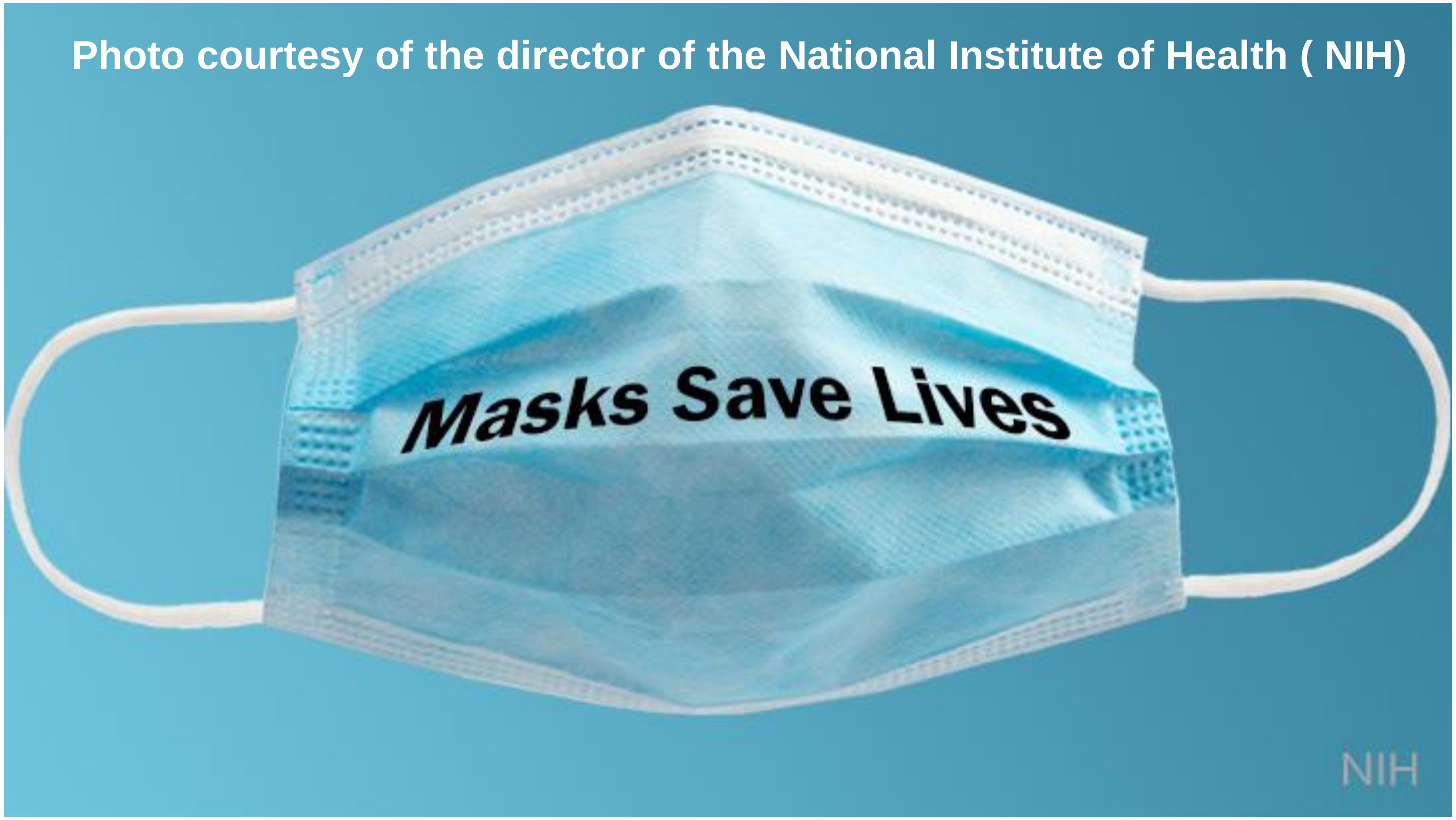
EE 330
Lecture 20

Bipolar Device Modeling

Exam Schedule

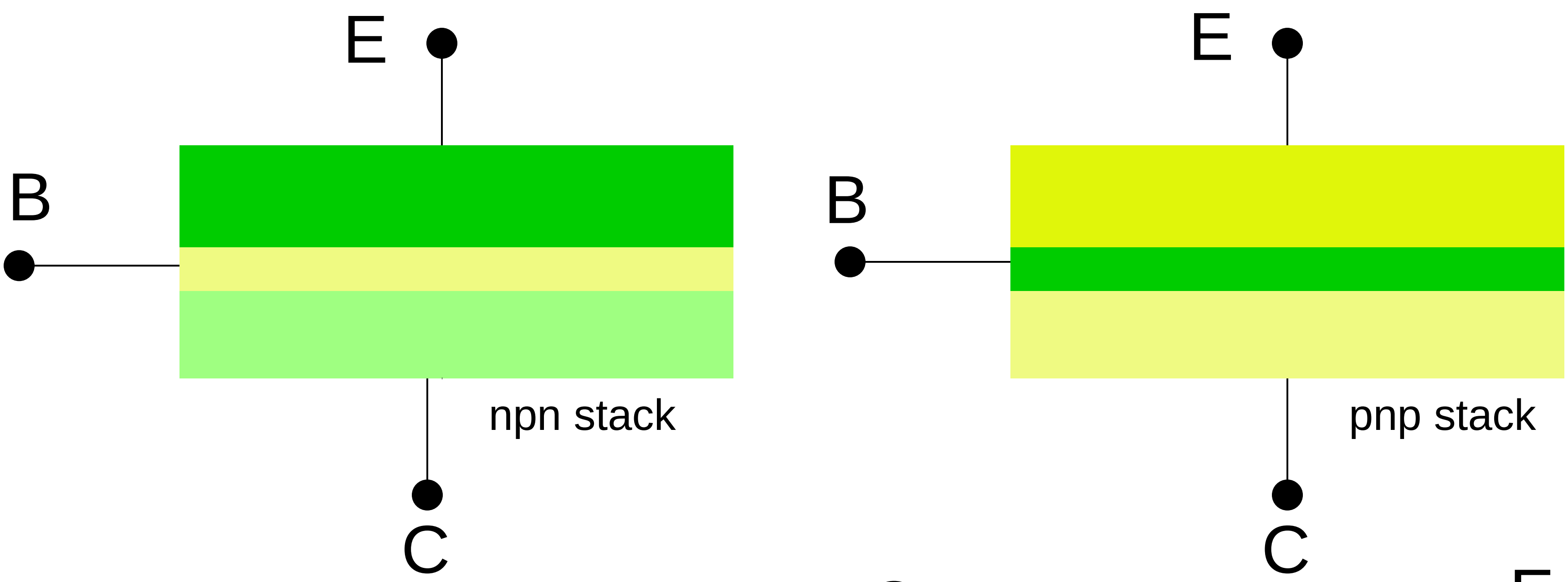
Exam 1	Friday Sept 24
Exam 2	Friday Oct 22
Exam 3	Friday Nov 19
Final	Tues Dec 14 12:00 p.m.



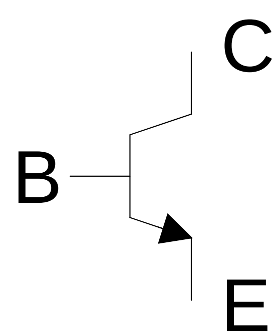
As a courtesy to fellow classmates, TAs, and the instructor

Wearing of masks during lectures and in the laboratories for this course would be appreciated irrespective of vaccination status

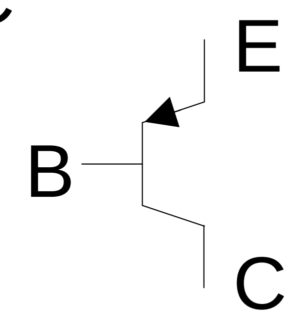
Bipolar Transistors



- **Bipolar Devices Show Basic Symmetry**
- **Electrical Properties not Symmetric**
- **Designation of C and E critical**



npn transistor

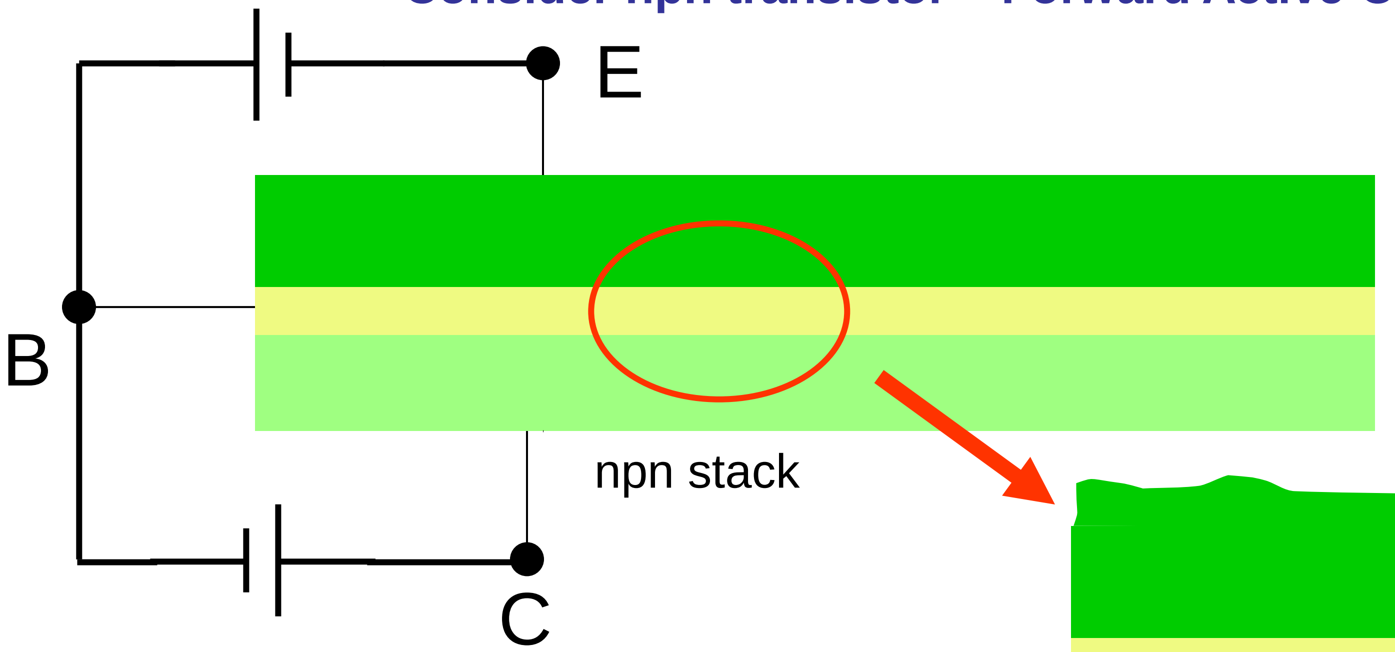


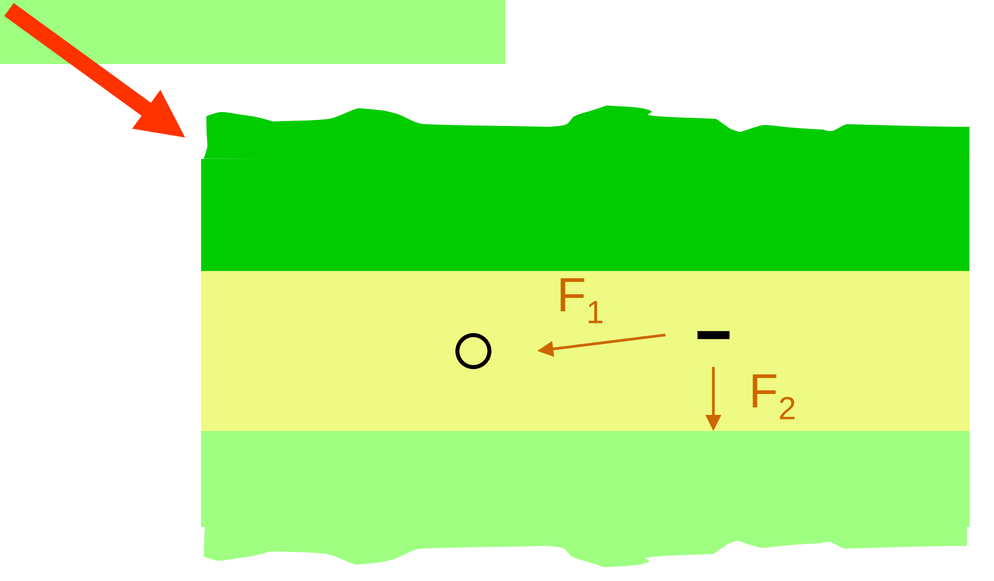
pnp transistor

With proper doping and device sizing these form Bipolar Transistors

Bipolar Operation

Consider npn transistor – Forward Active Operation





So, what will happen?

Some will recombine with holes and contribute to base current and some will be attracted across BC junction and contribute to collector

Size and thickness of base region and relative doping levels will play key role in percent of minority carriers injected into base contributing to collector current

Simple dc model

npn transistor – Forward Active Operation

$$\left.\begin{aligned} I_B &= \tilde{I}_S e^{\frac{V_{BE}}{V_t}} \\ I_C &= \beta \tilde{I}_S e^{\frac{V_{BE}}{V_t}} \end{aligned}\right\} \longrightarrow \left.\begin{aligned} I_B &= \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}} \\ I_C &= J_S A_E e^{\frac{V_{BE}}{V_t}} \end{aligned}\right\}$$

$$V_t = \frac{kT}{q}$$

$k/q = 8.62 \times 10^{-5}$

$$V_t = \frac{kT}{q}$$

J_S is termed the saturation current density

Process Parameters : J_S, β

Design Parameters: A_E

Environmental parameters and physical constants: k,T,q

At room temperature, V_t is around 26mV

J_S very small – around .25fA/u^2 at room temperature

Simple dc model

npn transistor – Forward Active Operation

$$I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}$$

$$I_C = J_S A_E e^{\frac{V_{BE}}{V_t}}$$

$$V_t = \frac{kT}{q}$$

As with the diode, the parameter J_S is highly temperature dependent

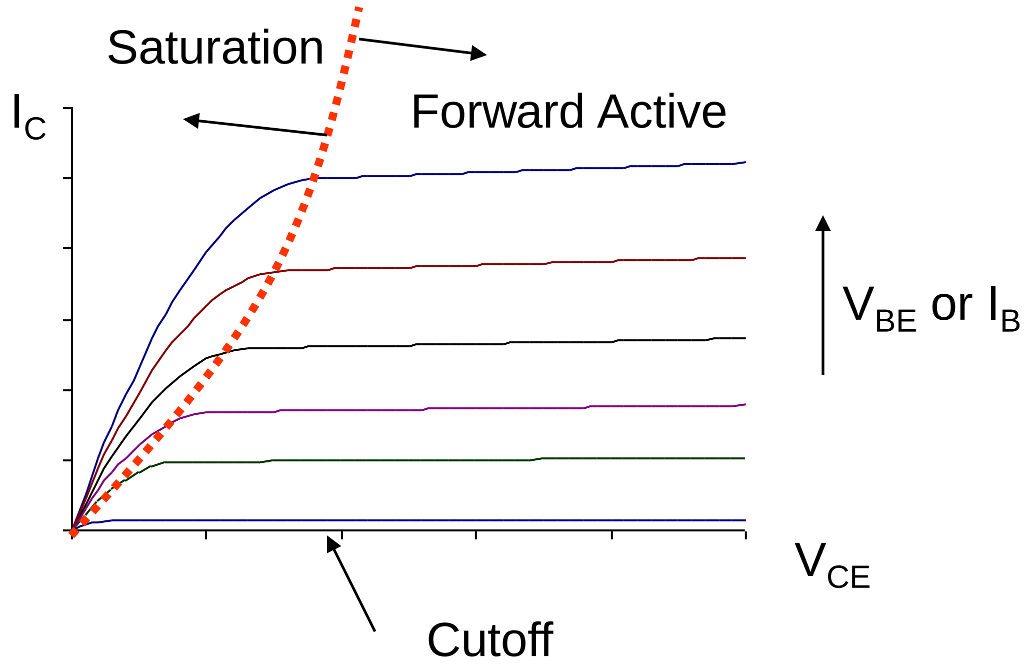
$$J_S = J_{SX}\left[T^m e^{\frac{-V_{G0}}{V_t}}\right]$$

Typical values for parameters: J_{SX}=20mA/μ^2, V_{G0}=1.17V, m=2.3

The parameter β is also somewhat temperature dependent but much weaker temperature dependence than J_{SX}.

Simple dc model

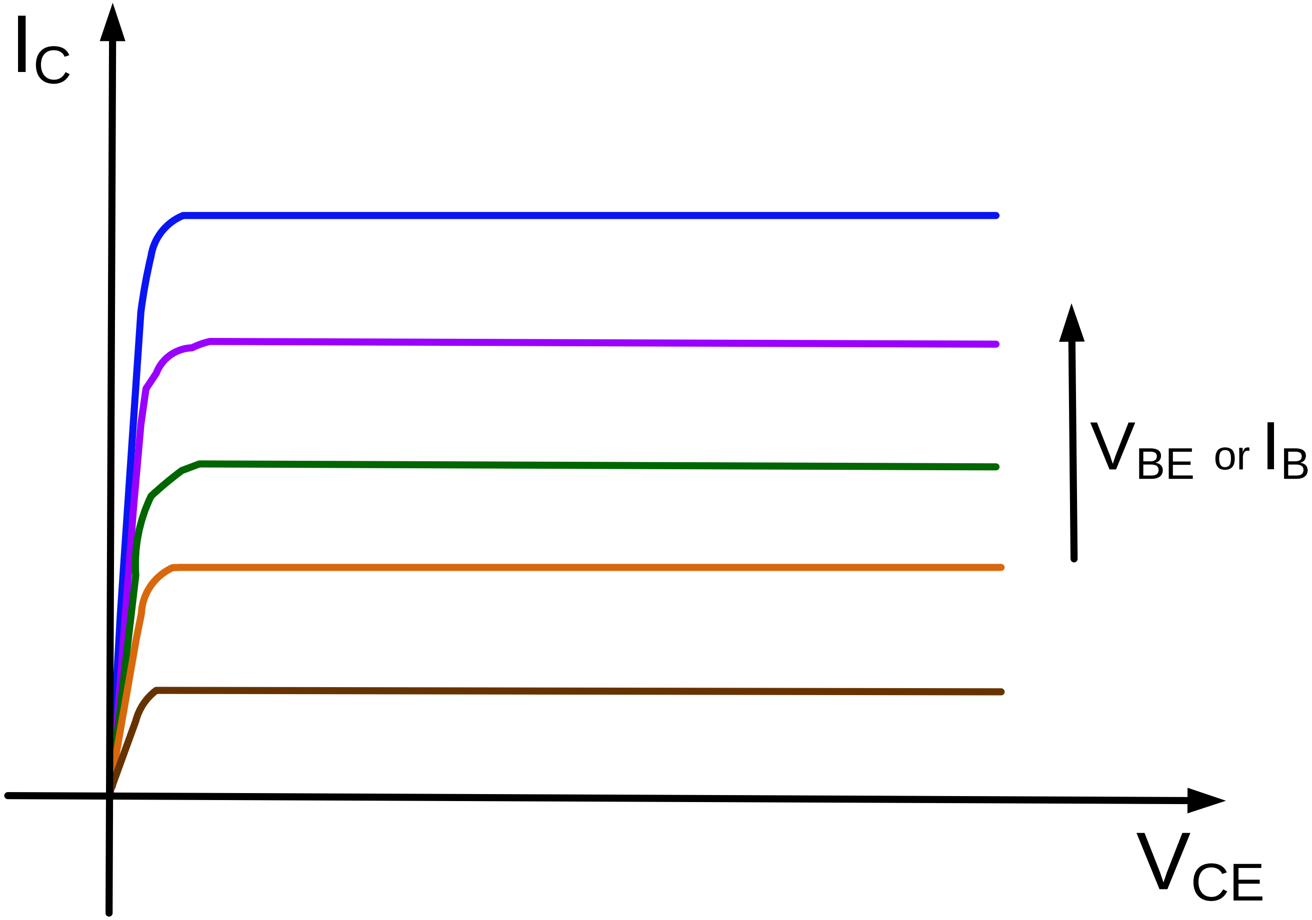
Typical Output Characteristics



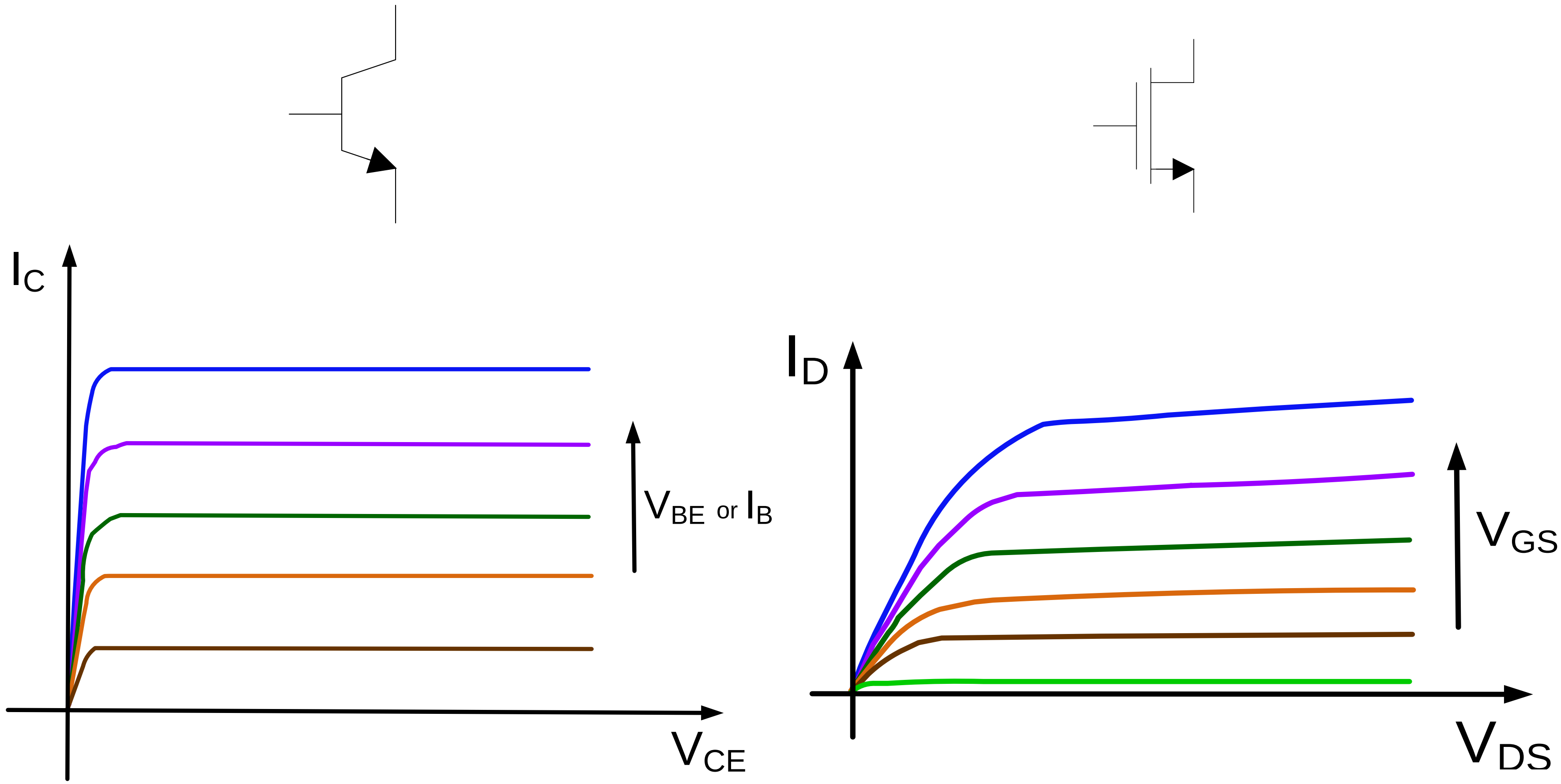
Forward Active region of BJT is analogous to Saturation region of MOSFET

Saturation region of BJT is analogous to Triode region of MOSFET

Better Model of Output Characteristics

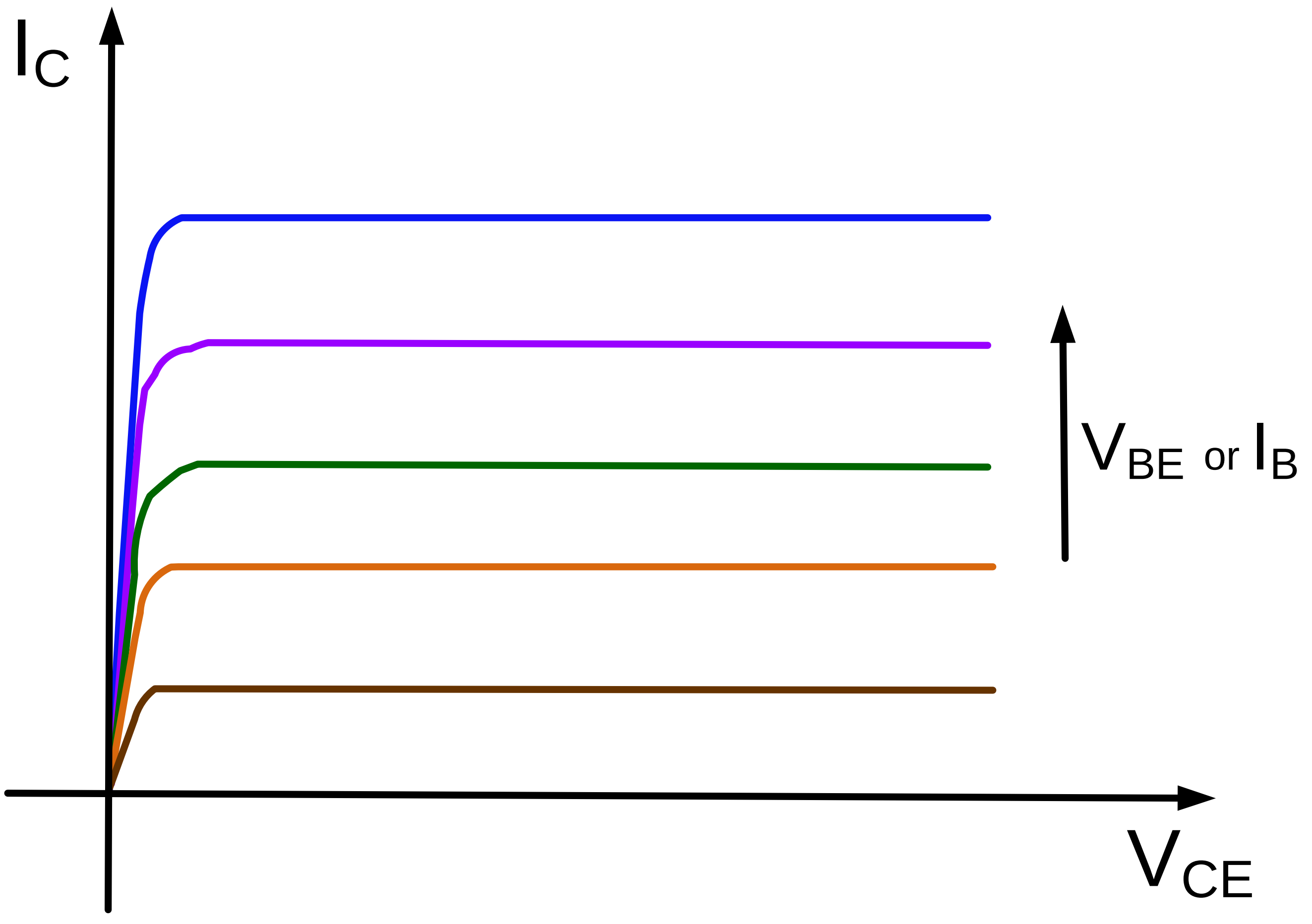


BJT and MOSFET Comparison



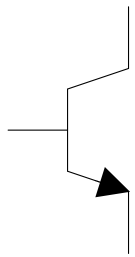
- Same general characteristics
- Spacings a bit different (Exponetial vs square law)
- Slope steeper for small V_{CE} compared to V_{DS}

Better Model of Output Characteristics

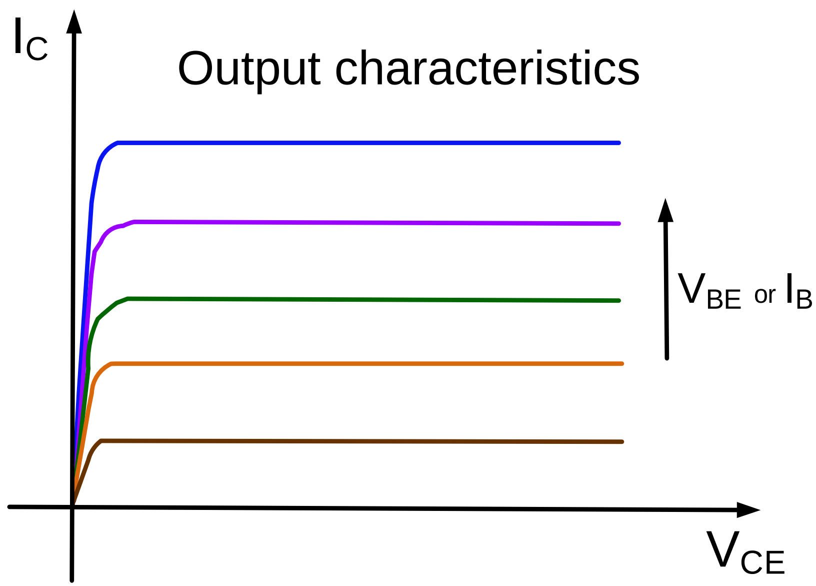


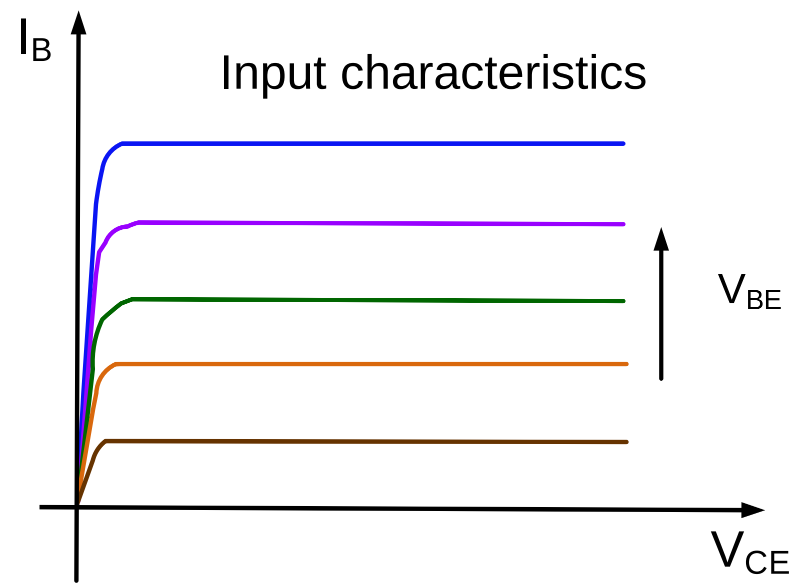
With scaled V_{CE} axis, transition in saturation very steep

BJT Model



$I_B = f_1(V_{BE}, V_{CE})$
$I_C = f_2(V_{BE}, V_{CE})$



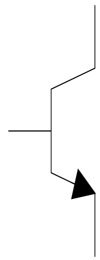


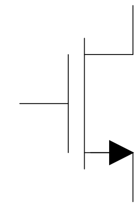
Require two graphical representations though vertical axis scales different by factor of β

Since $I_B=f(V_{BE})$, can use independent (V_{BE}) or dependent (I_B) variable for 2-D visualization of 3-dimensional I_C function

BJT and MOSFET Comparison

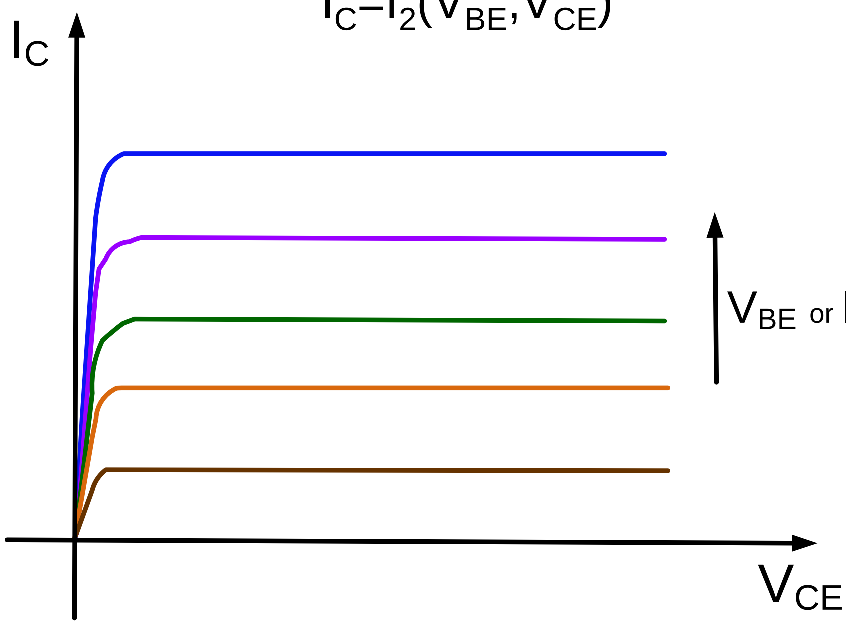
Output Characteristics

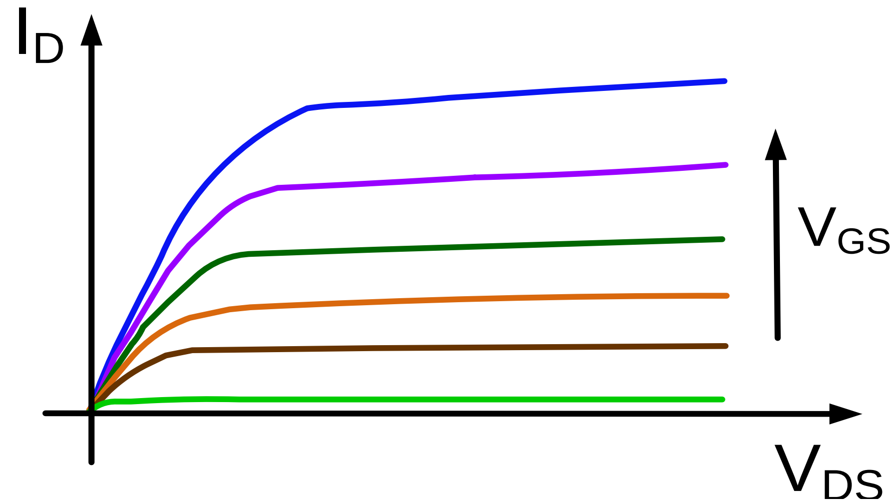




$I_C = f_2(V_{BE}, V_{CE})$

$I_D = f_{2M}(V_{GS}, V_{DS})$

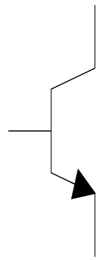


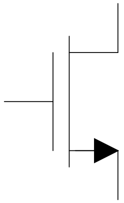


- Same general characteristics
- Spacings a bit different (Exponetial vs square law)
- Slope steeper for small V_{CE} compared to small V_{DS}

BJT and MOSFET Comparison

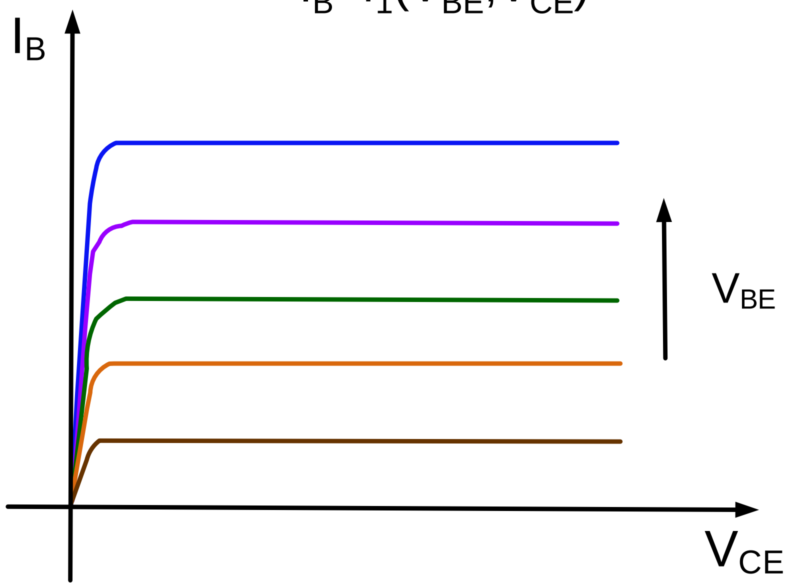
Input Characteristics

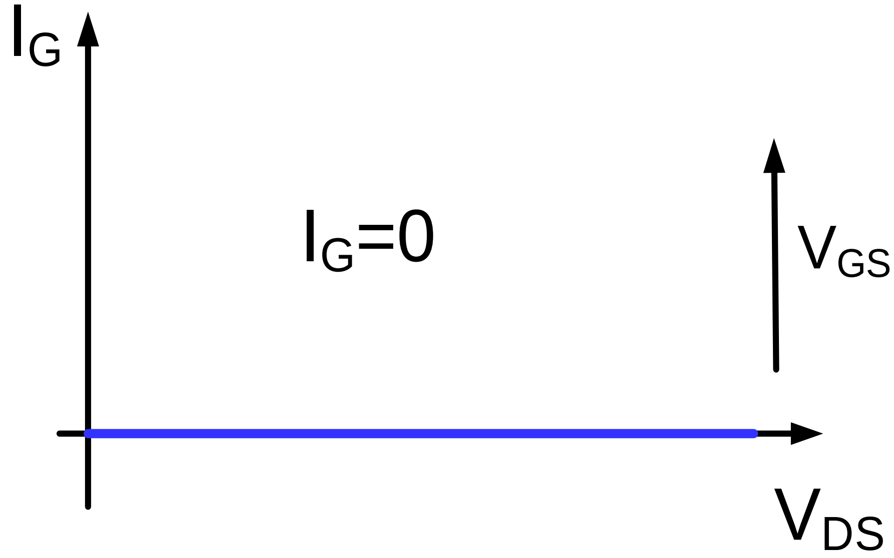




$I_B=f_1(V_{BE},V_{CE})$

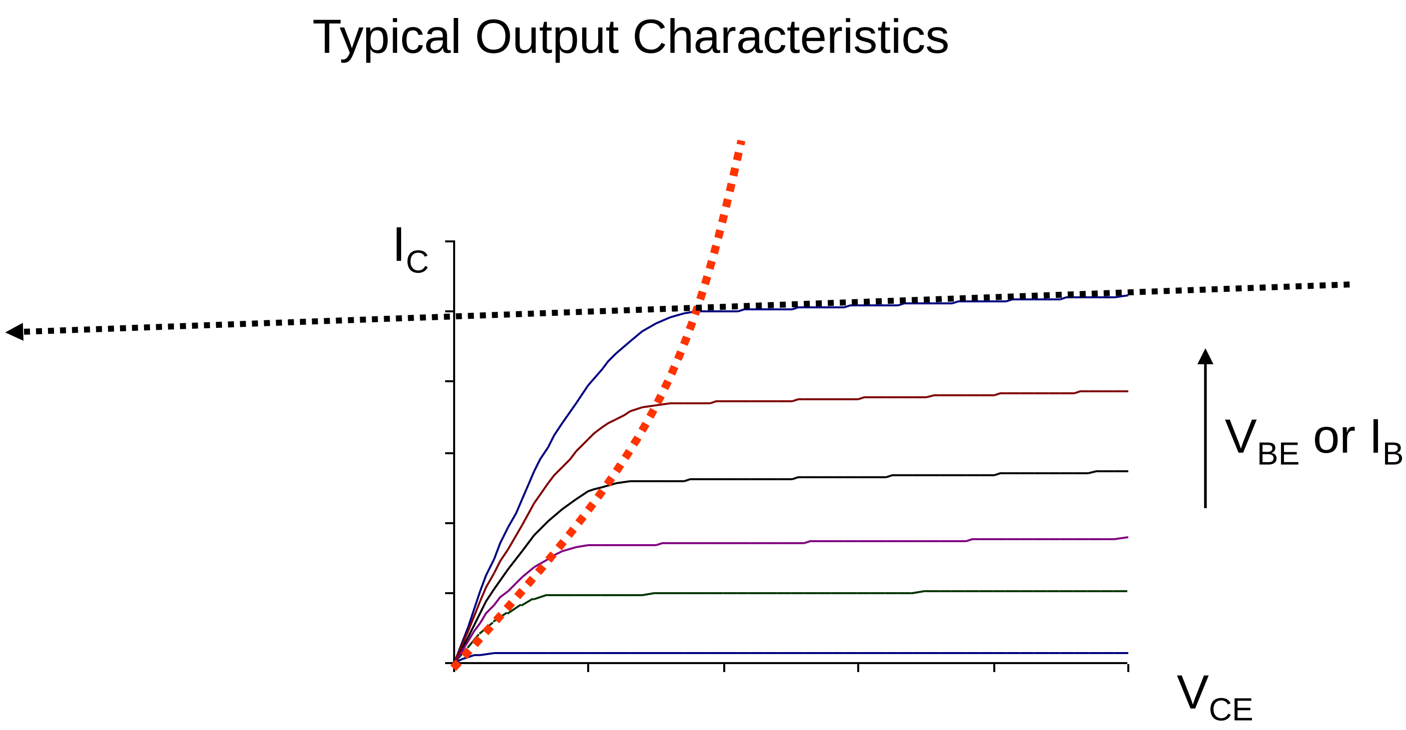
$I_G=f_{1M}(V_{GS},V_{DS})$





Did not need to graphically show input characteristics for MOS transistors since $I_G=0$

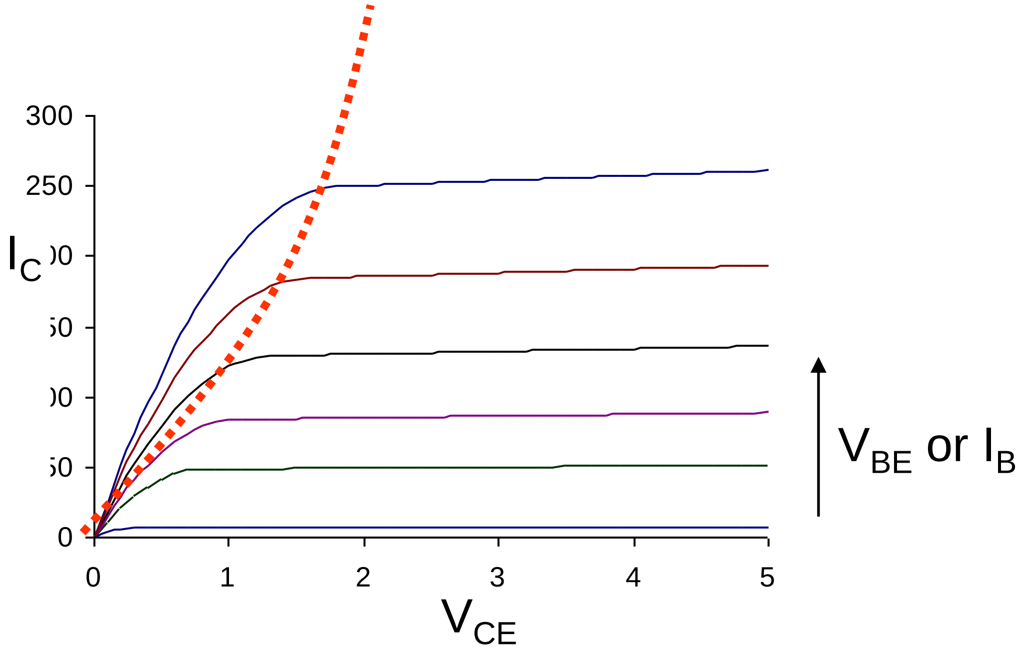
Improved simple dc model



- Projections of these tangential lines all intercept the $-V_{CE}$ axis at the same place and this is termed the Early voltage, V_{AF} (actually $-V_{AF}$ is intercept)
- Typical values of V_{AF} are in the 100V to 200V range
- Can multiply expression for I_C in Forward Active Region by term $\left(1+\frac{V_{CE}}{V_{AF}}\right)$ to account for slope

Improved simple dc model

(graphically showing only output characteristics)



$$I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}$$

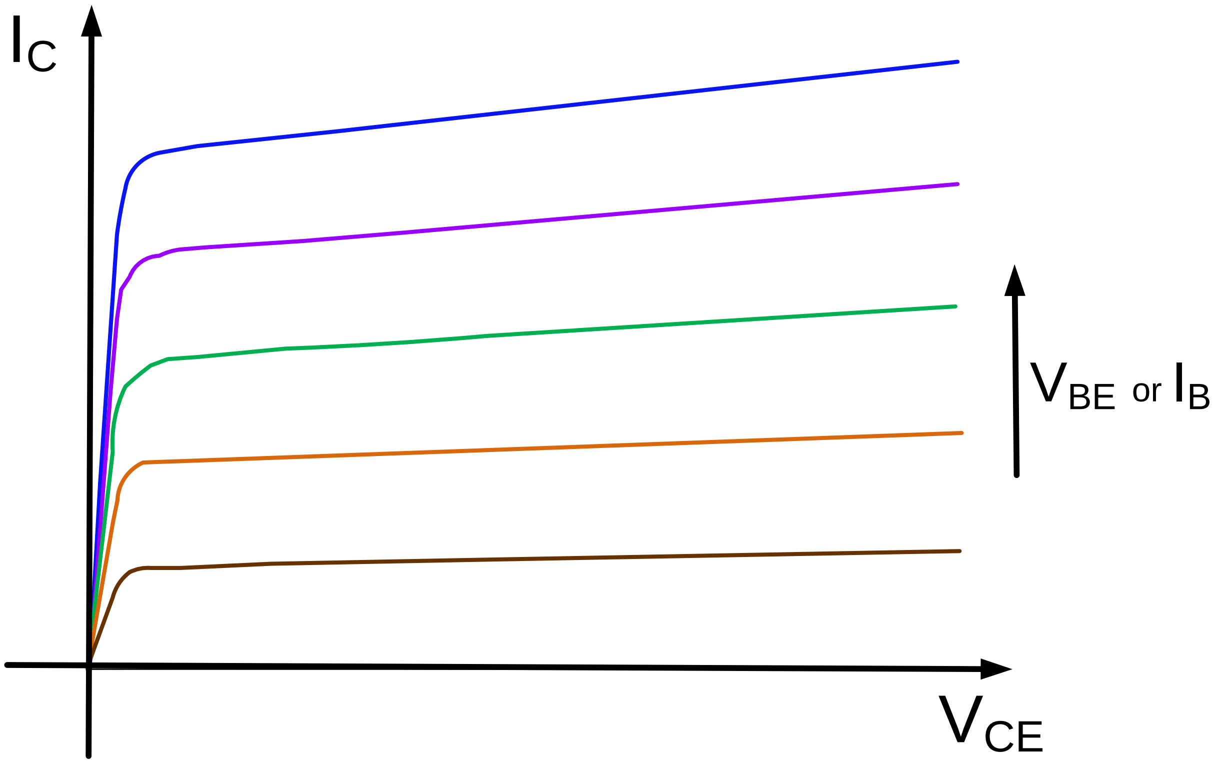
$$I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \left(1 + \frac{V_{CE}}{V_{AF}}\right)$$

Valid only in Forward Active Region

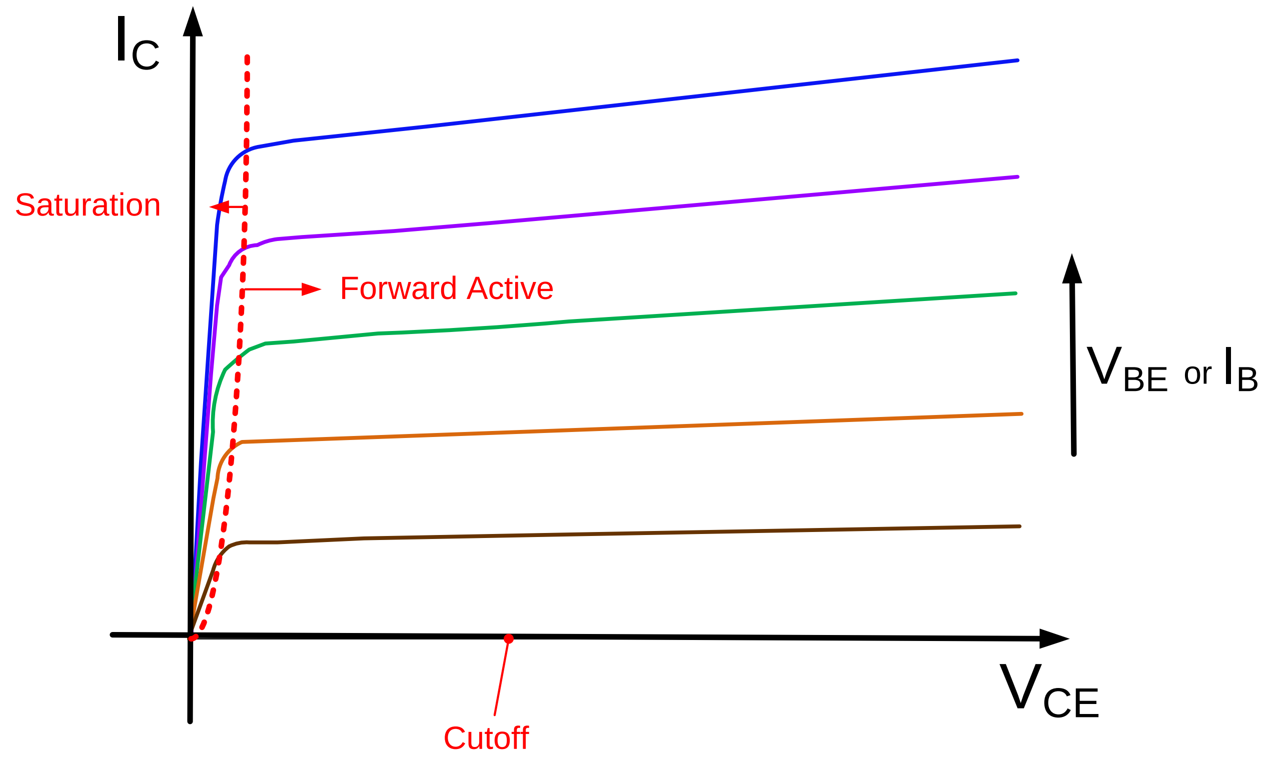
Need models in saturation and cutoff regions

Improved simple BJT dc model

Typical Output Characteristics



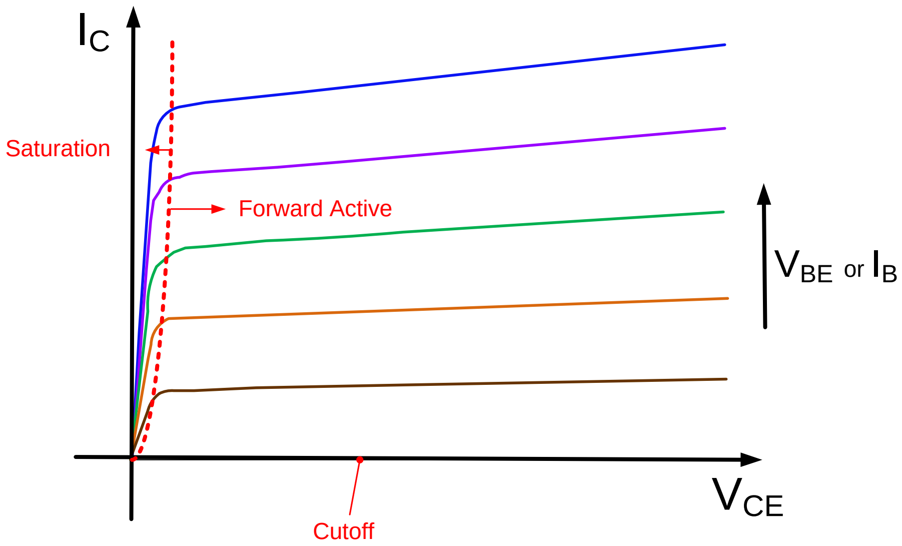
Improved simple BJT dc model

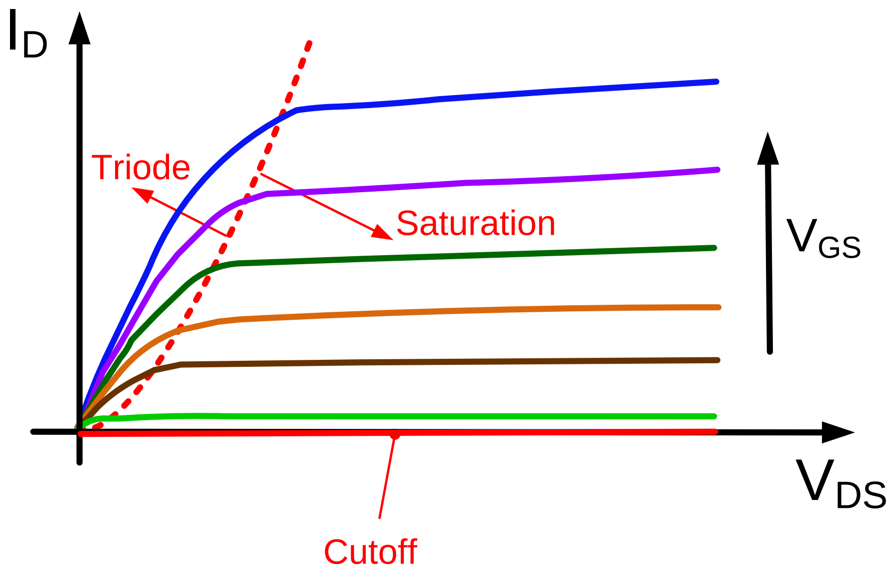


Need analytical models in saturation and cutoff regions

Improved simple BJT dc model

Typical Output Characteristics





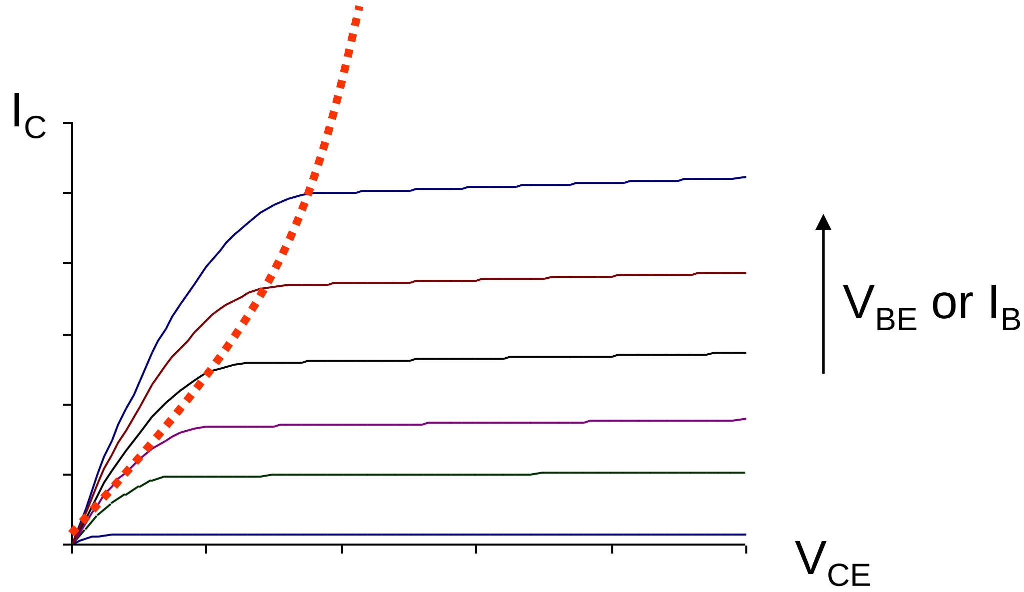
Recall:

Forward Active region of BJT is analogous to Saturation region of MOSFET

Saturation region of BJT is analogous to Triode region of MOSFET

Improved dc model

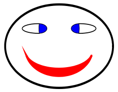
(graphically showing only output characteristics)



$$V_t = \frac{kT}{q}$$

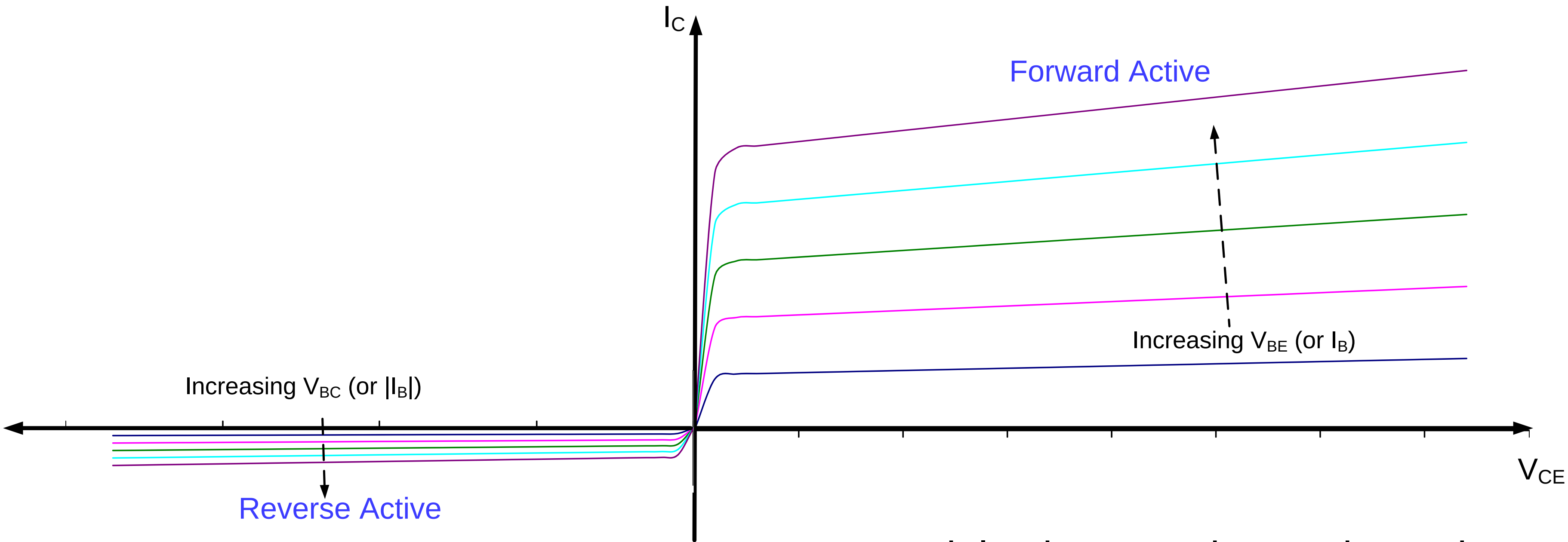
$$I_E = -\frac{J_S A_E}{\alpha_F}\left(e^{\frac{V_{BE}}{V_t}} - 1\right) + J_S A_E\left(e^{\frac{V_{BC}}{V_t}} - 1\right)$$

$$I_C = J_S A_E\left(e^{\frac{V_{BE}}{V_t}} - 1\right) - \frac{J_S A_E}{\alpha_R}\left(e^{\frac{V_{BC}}{V_t}} - 1\right)$$

- Valid in All regions of operation 
- V_{AF} effects can be added
- Not mathematically easy to work with 
- Note dependent variables changes $\{I_E, I_C\}$
- Termed Ebers-Moll model
- Reduces to previous model in FA region
- Little use in Reverse Active Region

Improved dc model

(graphically showing only output characteristics)



$$V_t = \frac{kT}{q}$$

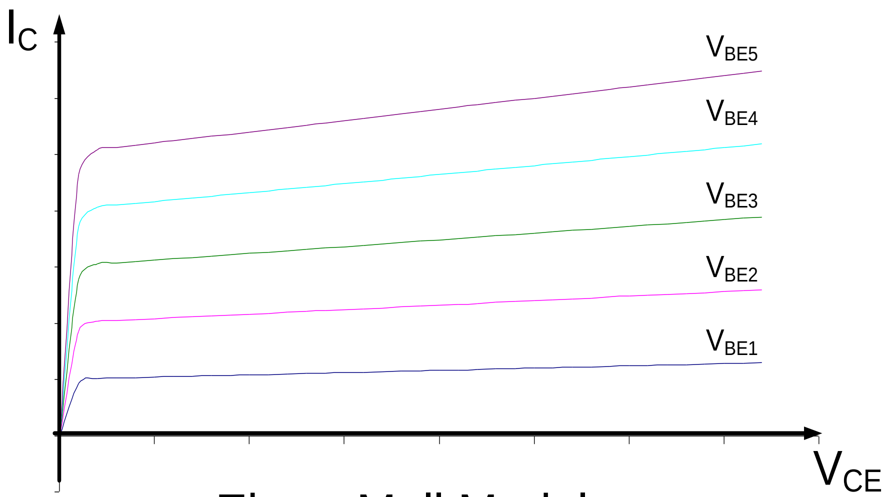
$$I_E = -\frac{J_S A_E}{\alpha_F}\left(e^{\frac{V_{BE}}{V_t}} - 1\right) + J_S A_E\left(e^{\frac{V_{BC}}{V_t}} - 1\right)$$

$$I_C = J_S A_E\left(e^{\frac{V_{BE}}{V_t}} - 1\right) - \frac{J_S A_E}{\alpha_R}\left(e^{\frac{V_{BC}}{V_t}} - 1\right)$$

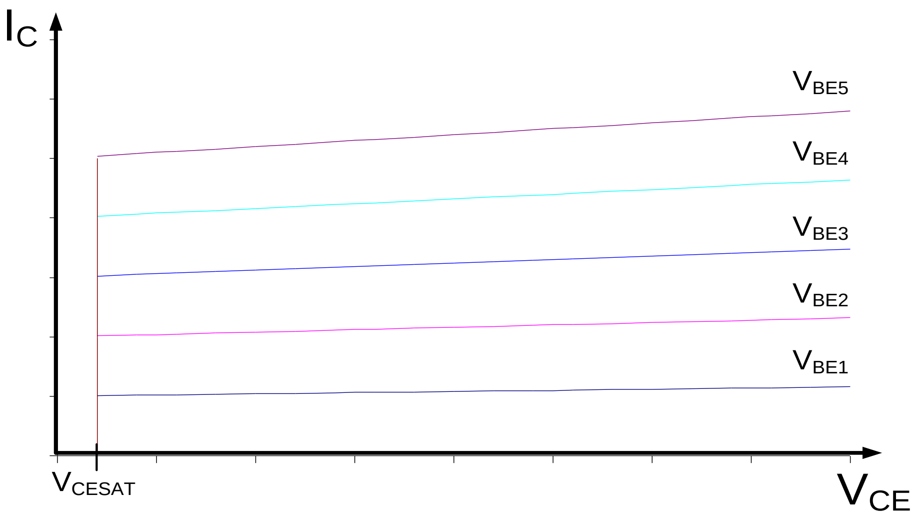
- Model using I_E and I_C as dependent variables
- Valid in All regions of operation
- V_{AF} effects can be added
- Not mathematically easy to work with
- Note dependent variables changes
- Termed Ebers-Moll model
- Reduces to previous model in FA region
- Little use in Reverse Active Region

Simplified Multi-Region Model

(graphically showing only output characteristics)



Ebers-Moll Model



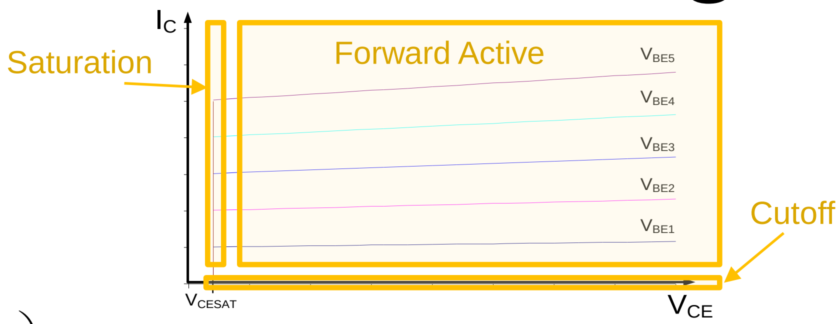
Simplified Multi-Region Model

- Observe V_{CE} around 0.2V when saturated
- V_{BE} around 0.6V when saturated
- In most applications, exact V_{CE} and V_{BE} voltage in saturation not critical

Simplified model in saturation:

$$\left.\begin{aligned} V_{BE}&=0.7V \\ V_{CE}&=0.2V \end{aligned}\right\} \quad \text{Saturation}$$

Simplified Multi-Region Model



$$I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \left(1 + \frac{V_{CE}}{V_{AF}}\right)$$

$$I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}} \qquad V_t = \frac{kT}{q}$$

Forward Active

V_{BE}=0.7V
V_{CE}=0.2V

Saturation

I_C=I_B=0

Cutoff

- This is a piecewise model suitable for analytical calculations
- Can easily extend to reverse active mode but of little use
- Still need conditions for operating in the 3 regions !!

Simplified Multi-Region Model

"Forward" Regions : $\beta=\beta_F$

	Conditions	
$I_C = J_S A_E e^{\frac{V_{BE}}{V_t}}\left(1+\frac{V_{CE}}{V_{AF}}\right)$ $I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}$	$V_{BE}>0.4V$ $V_{BC}<0$	Forward Active
$V_{BE}=0.7V$ $V_{CE}=0.2V$	$I_C<\beta I_B$	Saturation
$I_C=I_B=0$	$V_{BE}<0$ $V_{BC}<0$	Cutoff

Process Parameters: $\{J_S, \beta, V_{AF}\}$ $V_t = \frac{kT}{q}$ Design Parameters: $\{A_E\}$

- Process parameters highly process dependent
- J_S highly temperature dependent as well, β modestly temperature dependent
- This model is dependent only upon emitter area, independent of base and collector area !
- Currents scale linearly with A_E and not dependent upon shape of emitter
- A small portion of the operating region is missed with this model but seldom operate in the missing region

Simplified Multi-Region Model

Alternate equivalent model

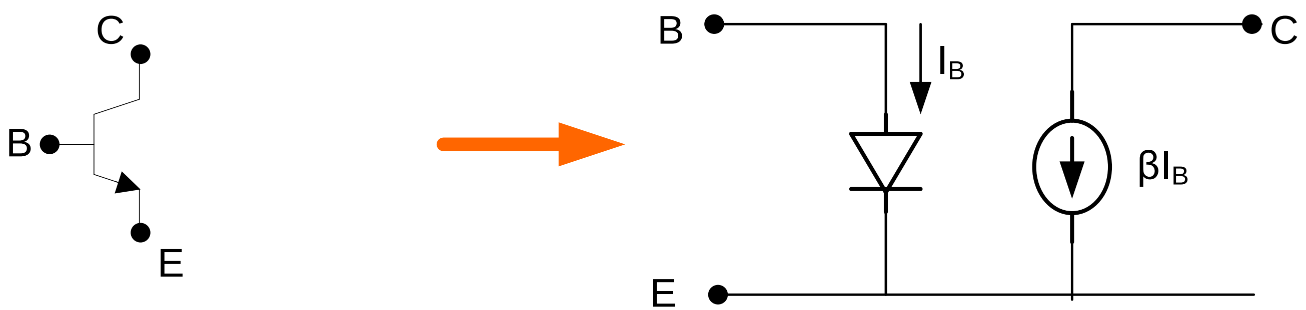
	Conditions	
$I_C = \beta I_B\left(1+\frac{V_{CE}}{V_{AF}}\right)$ $I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}$ $V_t = \frac{kT}{q}$	$V_{BE}>0.4V$ $V_{BC}<0$	Forward Active
$V_{BE}=0.7V$ $V_{CE}=0.2V$	$I_C<\beta I_B$	Saturation
$I_C=I_B=0$	$V_{BE}<0$ $V_{BC}<0$	Cutoff

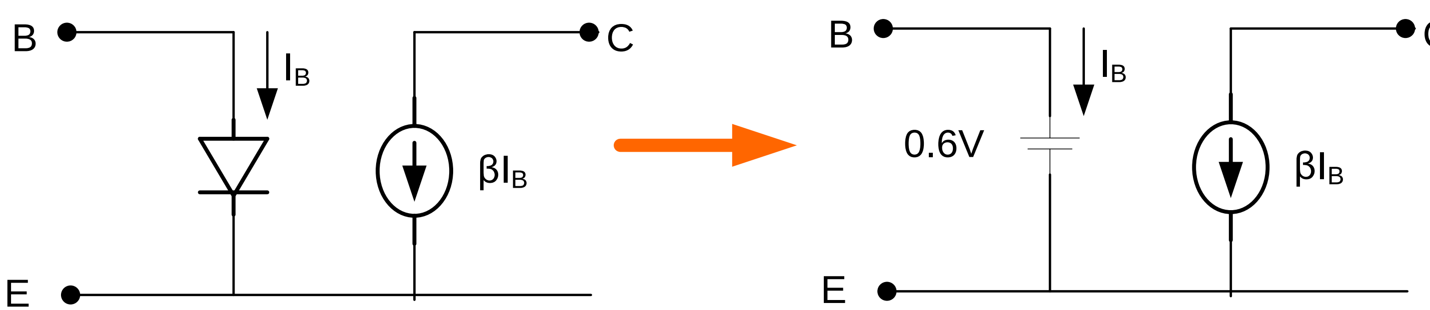
A small portion of the operating region is missed with this model but seldom operate in the missing region

Further Simplified Multi-Region dc Model

(by neglecting V_{AF})

Forward Active

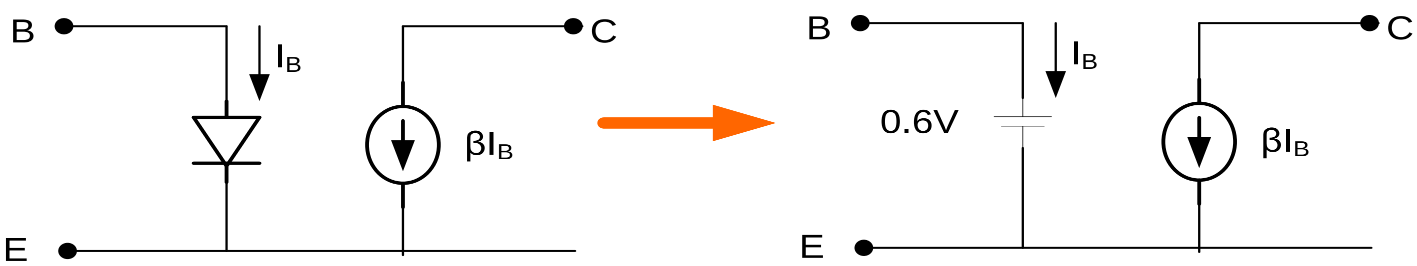




Adequate when it makes little difference whether V_{BE}=0.6V or V_{BE}=0.7V

Simplified Multi-Region dc Model

Forward Active

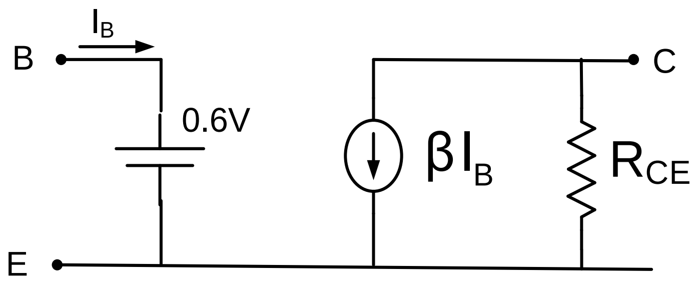


Mathematically

$V_{BE}=0.6V$
$I_C=\beta I_B$

Or, if want to show slope in I_C-V_{CE} characteristics

$V_{BE}=0.6V$
$I_C=\beta I_B(1+V_{CE}/V_{AF})$



$$R_{CE}=\frac{V_{AF}}{\beta I_{BQ}}$$

R_{CE} highly nonlinear

Further Simplified Multi-Region dc Model

Equivalent Further Simplified Multi-Region Model

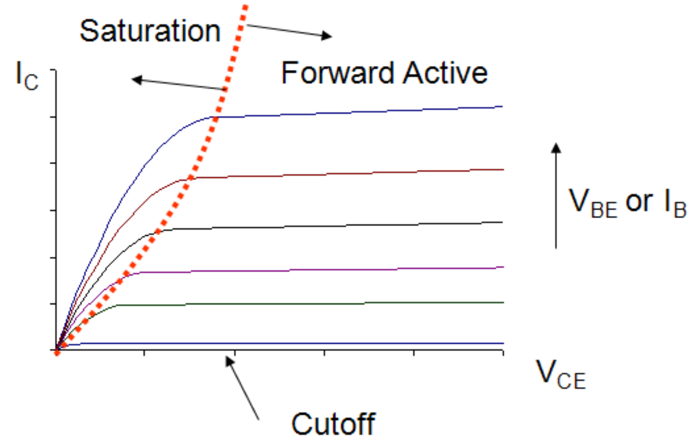
$I_C = \beta I_B$ $V_{BE} = 0.6V$ $V_t = \frac{kT}{q}$	$V_{BE}>0.4V$ $V_{BC}<0$	Forward Active
$V_{BE}=0.7V$ $V_{CE}=0.2V$	$I_C<\beta I_B$	Saturation
$I_C=I_B=0$	$V_{BE}<0$ $V_{BC}<0$	Cutoff

A small portion of the operating region is missed with this model but seldom operate in the missing region

Conditions for Regions of Operation in Multi-Region Model

$V_{BE}>0.4V$ $V_{BC}<0$	Forward Active
$I_C<\beta I_B$	Saturation
$V_{BE}<0$ $V_{BC}<0$	Cutoff

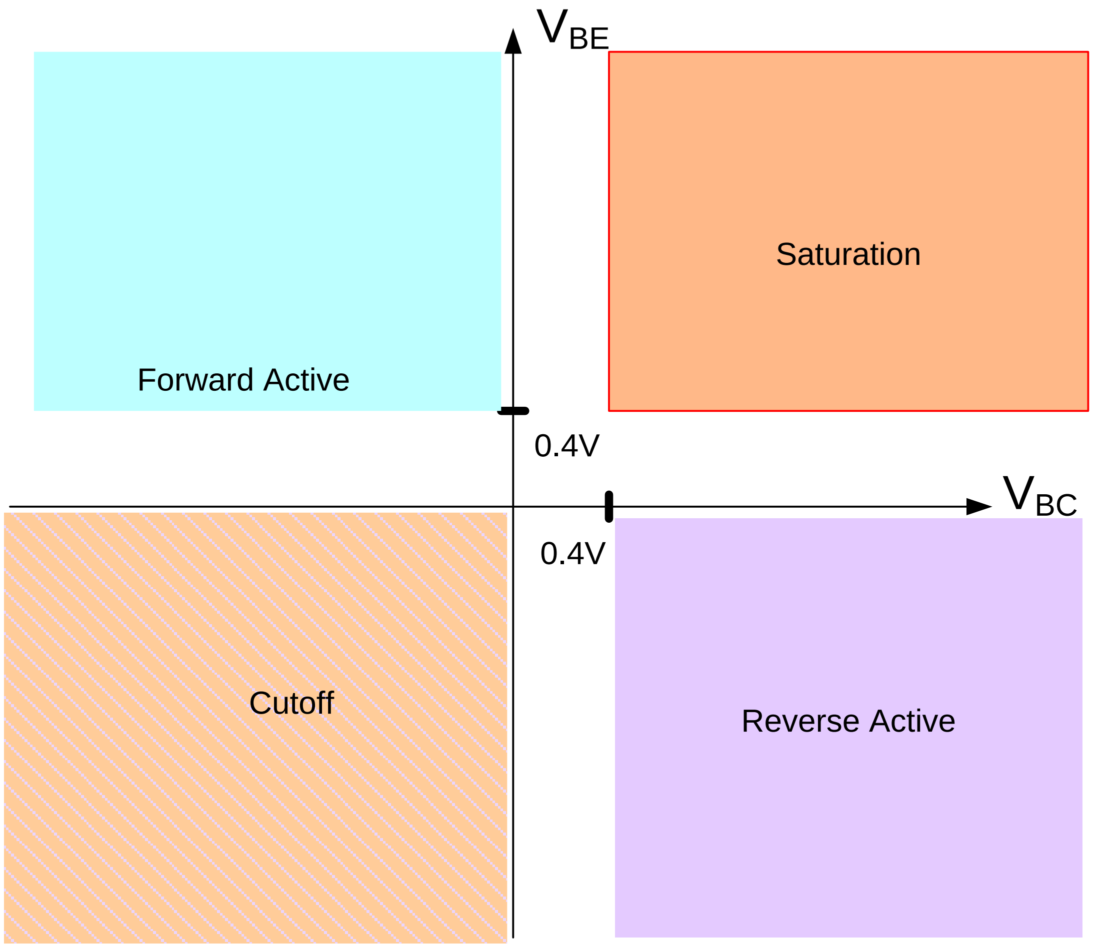
Note: One condition is on dependent variables !



Observe that in saturation, $I_C<\beta I_B$

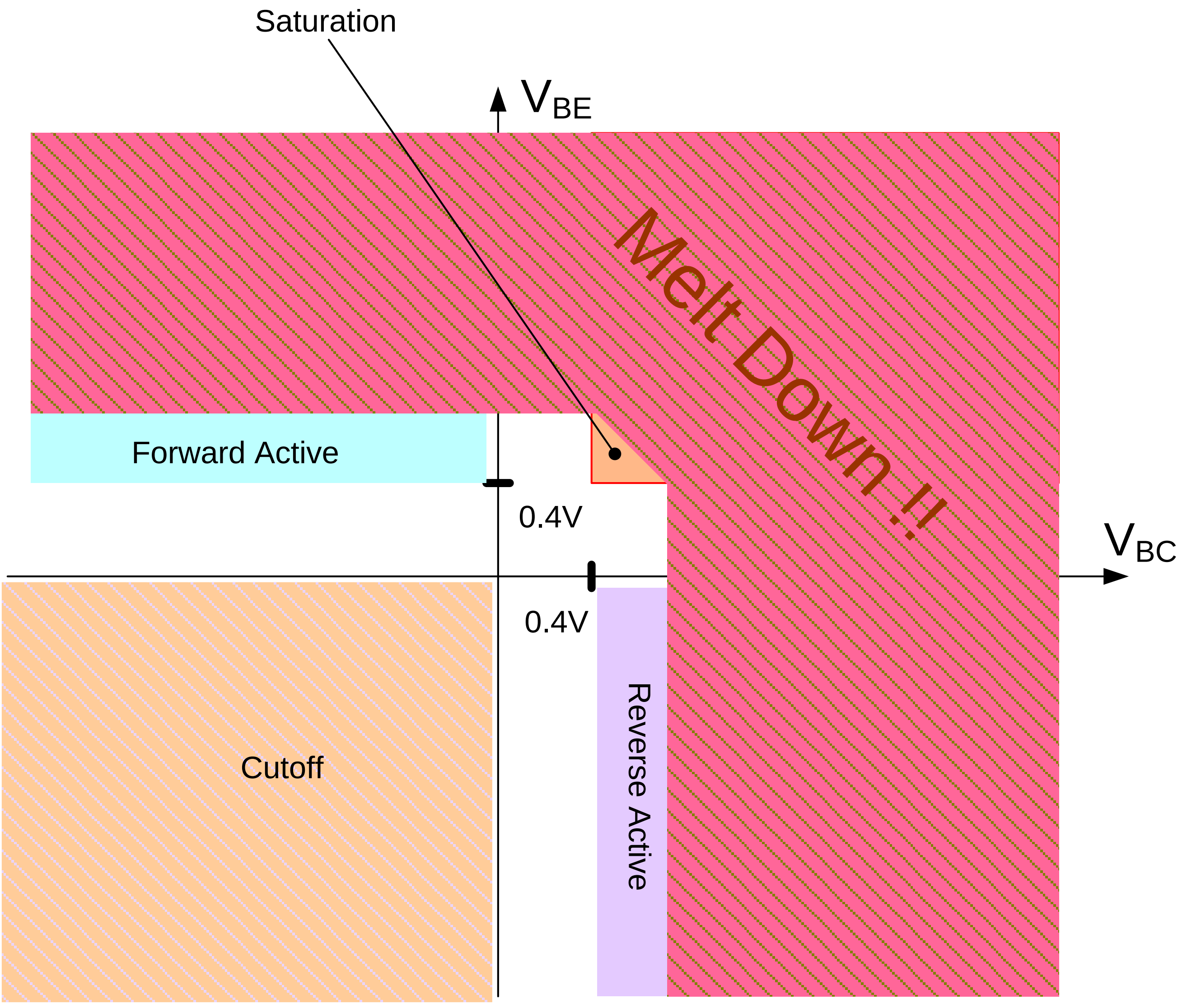
Can’t condition on independent variables in saturation because they are fixed in the model

Regions of Operation in Independent Parameter Domain used In multi-region models

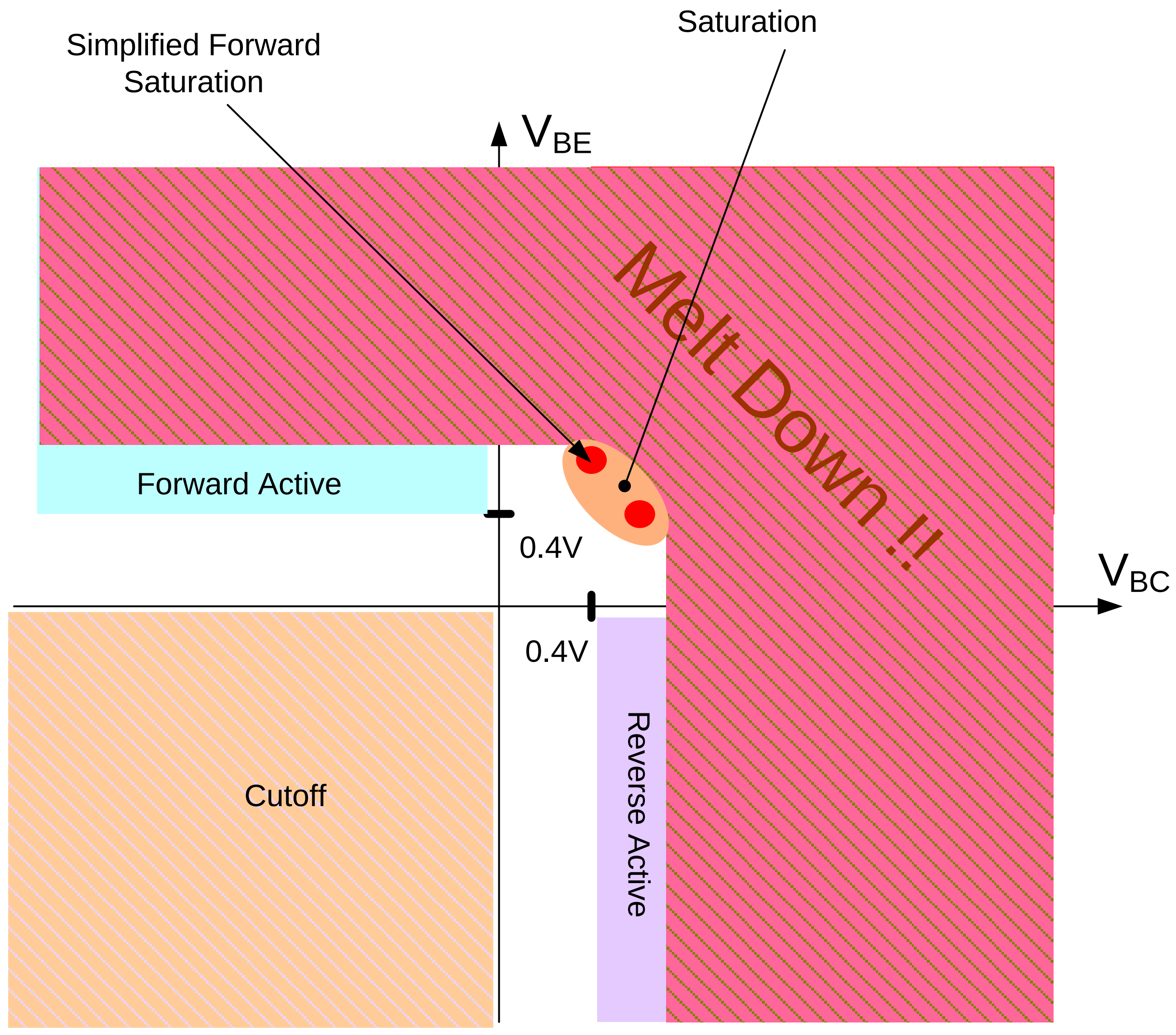


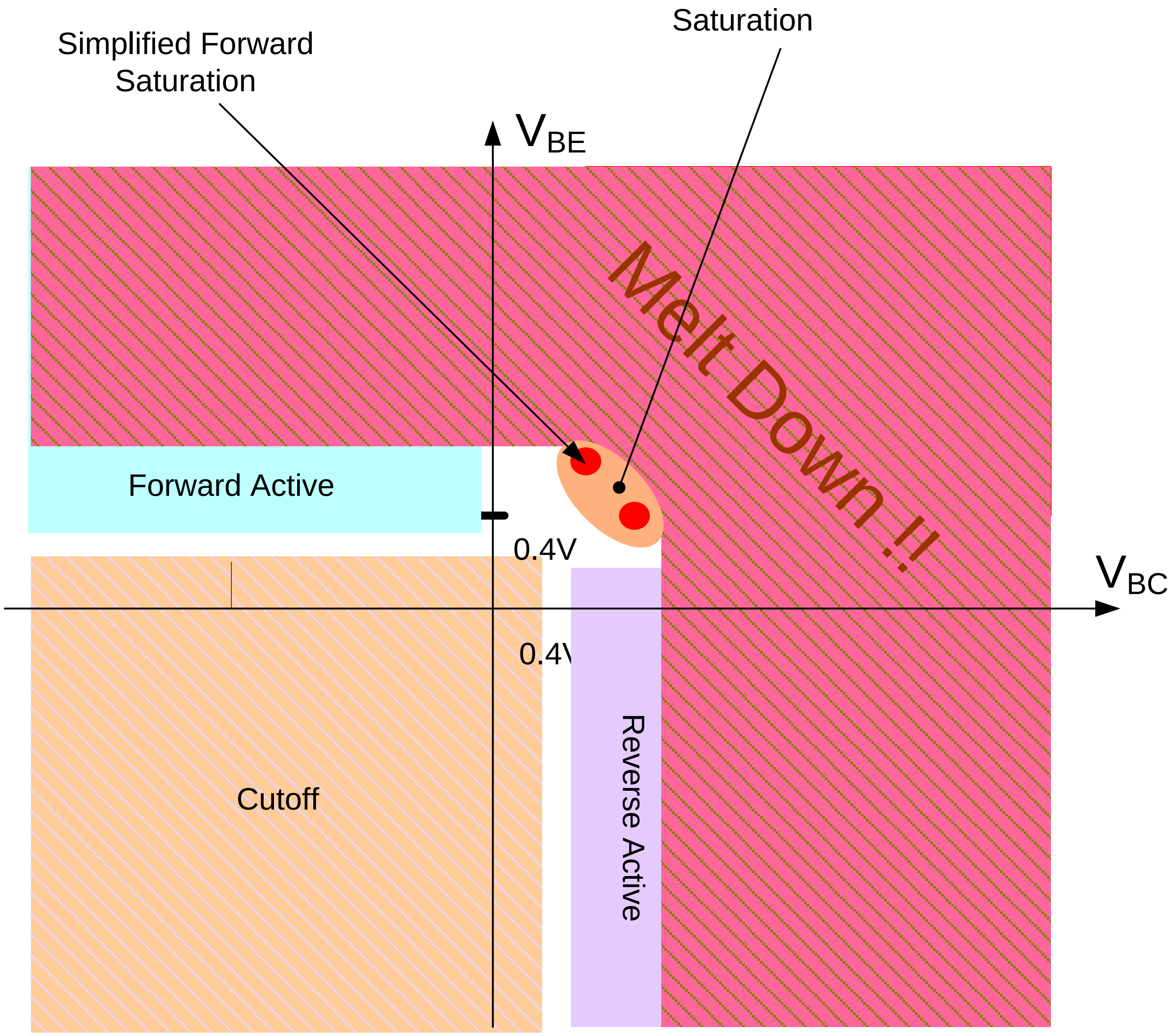
- Seldom operate in regions excluded in this picture
- Limited use in Reverse Active Mode

Excessive Power Dissipation if either junction has large forward bias



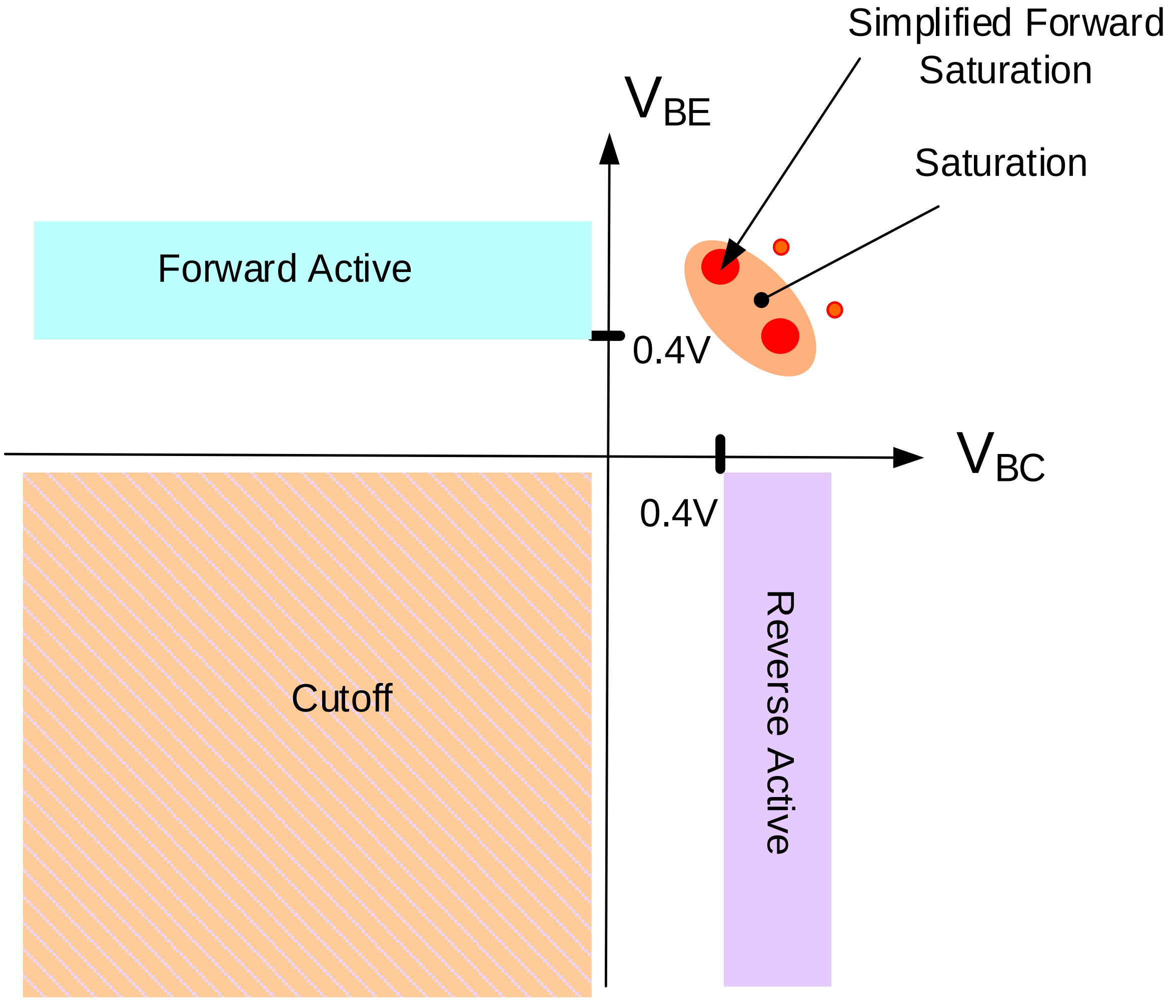
Safe regions of operation



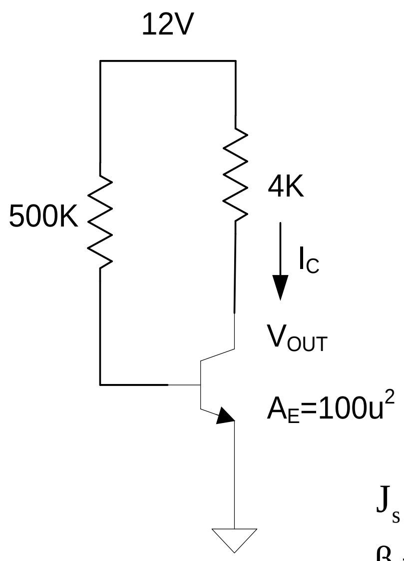


Actually cutoff, forward active, and reverse active regions can be extended modestly as shown and multi-region models still are quite good

Sufficient regions of operation for most applications



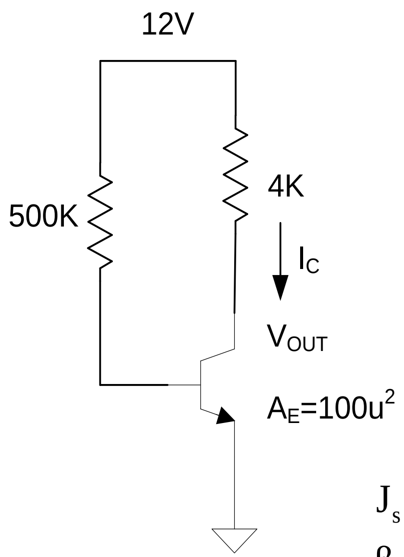
Example: Determine I_C and V_{OUT}



$$J_s = 10^{-16} A/\mu^2$$

$$\beta = 100$$

Example: Determine I_C and V_{OUT}

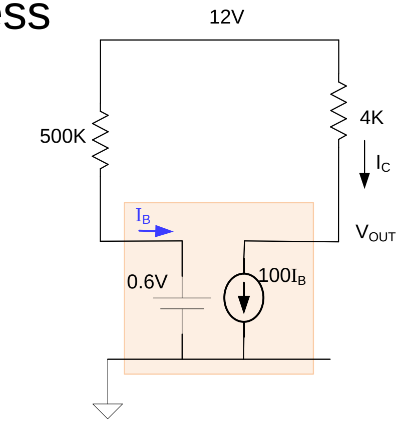


$J_s = 10^{-16} A/\mu^2$

$\beta = 100$

Solution:

1. Guess Forward Active Region (and model)
2. Solve Circuit with Guess
3. Verify model (if necessary)



$$I_B = \frac{(12-0.6)}{500K}$$

$$I_C = \beta I_B = 100\frac{(12-0.6)}{500K} = 2.28mA$$

$$V_{OUT} = 12 - I_C \bullet 4K = 2.88V$$

4. Verify FA Region

V_{BE}>0.4V and V_{BC}<0

$$V_{BE} = 0.6V \ > 0.4V$$

$$V_{BC} = 0.6V - 2.88V = -2.28V < 0$$

Verify Passes so solution is valid

$$I_C = 2.28mA$$

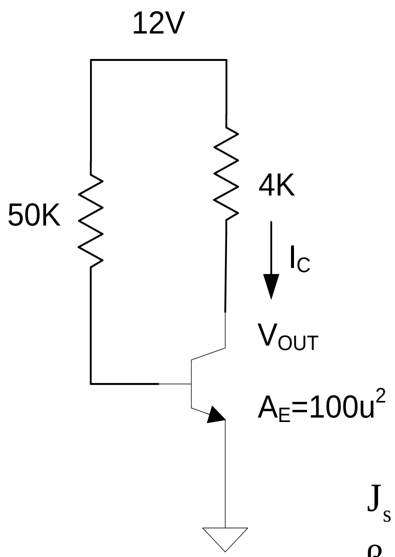
$$V_{OUT} = 2.88V$$

5. Verify model (if necessary)

Solve again with V_{BE}=0.7V
Will show V_{OUT} =2.96V so difference is small

Note solution independent of J_S and A_E

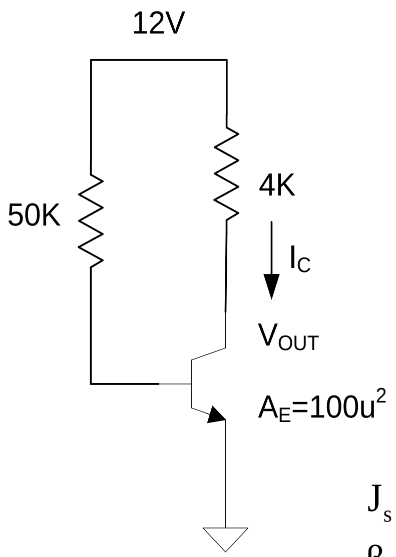
Example: Determine I_C and V_{OUT} ,



$$J_s = 10^{-16} A/\mu^2$$

$$\beta = 100$$

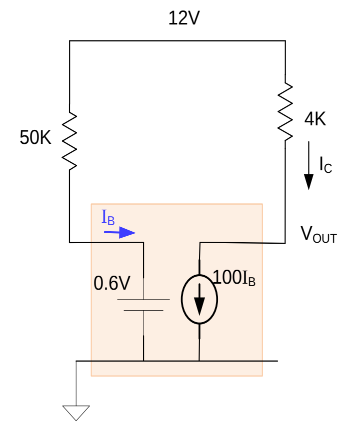
Example: Determine I_C and V_{OUT} .



$$J_s = 10^{-16} A/\mu^2$$
$$\beta = 100$$

Solution:

1. Guess Forward Active Region
2. Solve Circuit with Guess
3. Verify model (if necessary)



$$I_B = \frac{(12-0.6)}{50K}$$

$$I_C = \beta I_B = 100\frac{(12-0.6)}{50K} = 22.8mA$$

$$V_{OUT} = 12 - I_C \bullet 4K = -79.2V$$

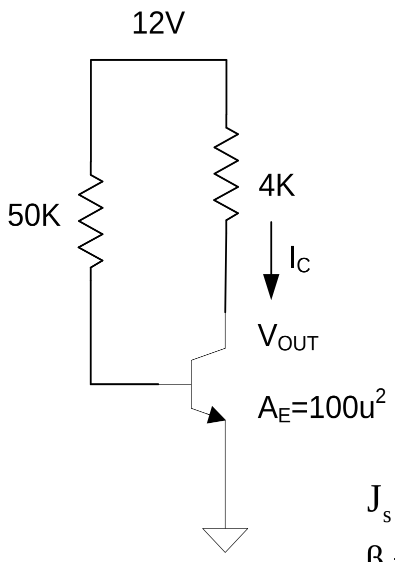
4. Verify FA Region $V_{BE}>0.4V$ and $V_{BC}<0$

$$V_{BE} = 0.6V \ > 0.4V$$

$$V_{BC} = 0.6V - -79.2V = +79.8V > 0$$

Verify Fails so solution is not valid

Example: Determine I_C and V_{OUT}

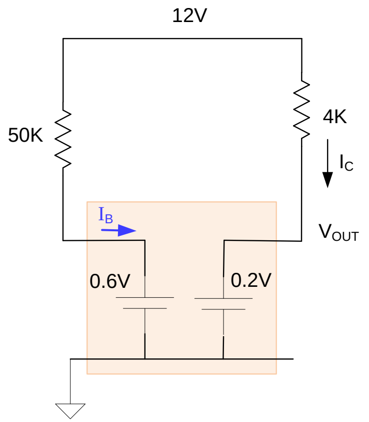


$J_s = 10^{-16} A/\mu^2$

$\beta = 100$

Solution:

5. Guess Saturation
6. Solve Circuit with Guess
7. Verify model (if necessary)



$$I_B = \frac{(12-0.6)}{50K} = 228\mu A$$

$$I_C = \frac{(12-0.2)}{4K} = 2.95mA$$

$$V_{OUT} = 0.2V$$

8. Verify SAT Region $I_C < \beta I_B$

$$\beta I_B = 100 \bullet 228\mu A = 22.8mA$$

$$I_C = 2.95mA$$

$$I_C = 2.95mA < \beta I_B = 22.8mA$$

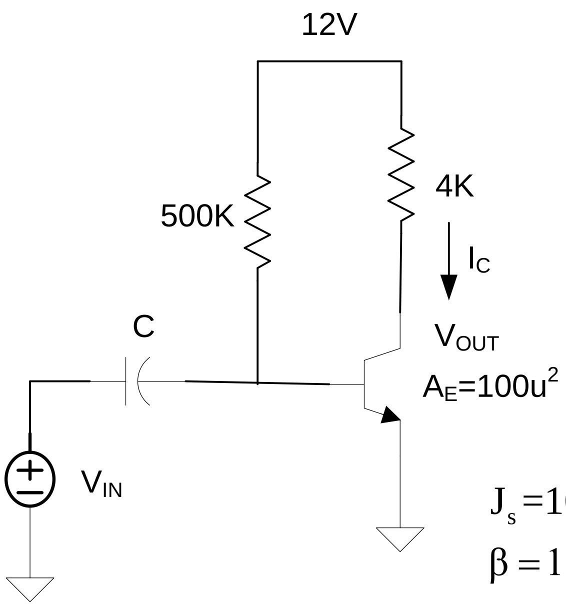
Verify SAT Passes so solution is valid

$$I_C = 2.95mA \qquad V_{OUT} = 0.2V$$

9. Verify model (if necessary)

(use V_{BE}=0.7V, no change in output)

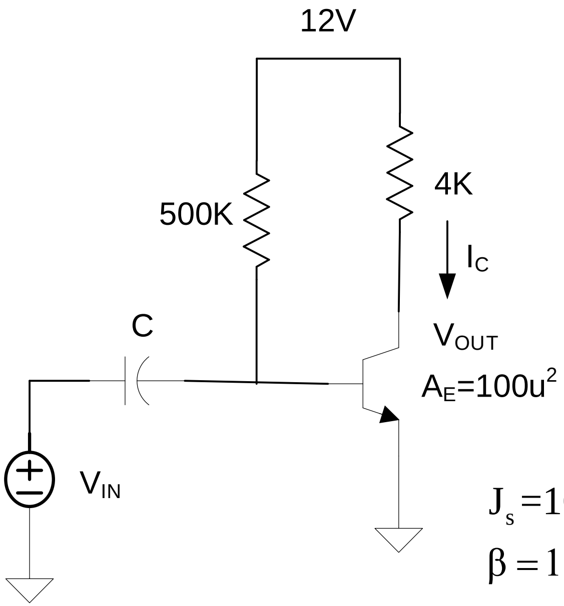
Example: Determine I_C and $V_{OUT.}$ Assume C is large and V_{IN} is very small.



$J_s = 10^{-16} A/\mu^2$

$\beta = 100$

Example: Determine I_C and $V_{OUT.}$ Assume C is large and V_{IN} is very small.



$J_s = 10^{-16} A/\mu^2$

$\beta = 100$

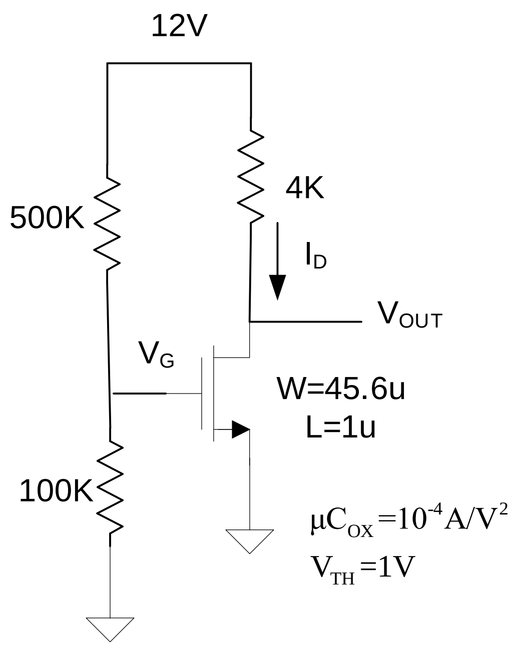
Solution:

Assume V_{IN}=0, then no current flows through C so circuit is identical to circuit of previous-previous example so

$$I_C = 2.28mA \qquad V_{OUT} = 2.88V$$

Note: If C is large and V_{IN} is small sinusoidal signal of sufficiently high frequency, the voltage across C will not change the input so V_{IN} is from an ac viewpoint coupled directly to base. In this case, the circuit will amplify V_{IN} and the gain will be very large due to the exponential relationship between I_C and V_{BE}.

Example: Determine I_D and $V_{OUT.}$ Assume C is large and V_{IN} is very small.



Solution:

Since I_G=0,

$$V_G = \frac{100K}{600K} 12V = 2V$$

Guess Saturation Region for MOSFET

$$V_{GS} > V_{TH} \quad V_{DS} > V_{GS} - V_{TH}$$

$$I_D = \mu C_{OX} \frac{W}{2L}(V_{GS} - V_{TH})^2$$

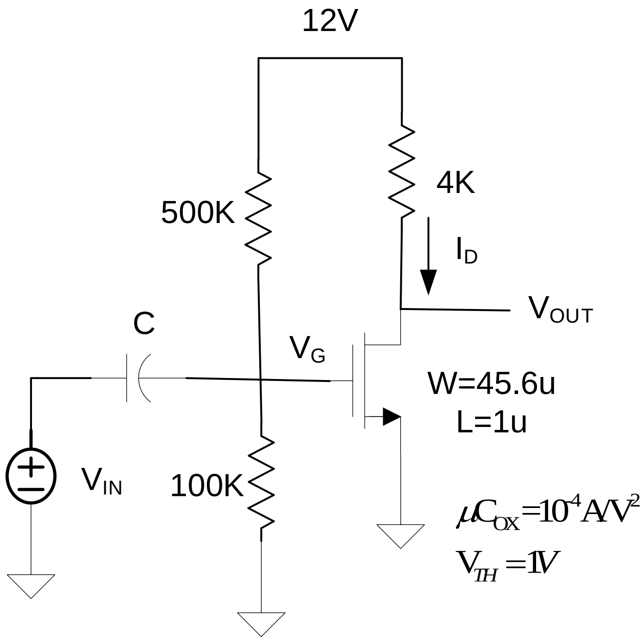
$$I_D = 10^{-4} \frac{45.6}{2}(2-1)^2 = 2.28mA$$

V_{OUT}=2.88V

Verify saturation $\quad 2V > 1V \quad 2.88V > 2V - 1V$

Note: solution dependent upon W,L,V_{TH}, and uC_{ox}

Example: Determine I_D and $V_{OUT.}$ Assume C is large and V_{IN} is very small.



Solution:

Assume V_{IN}=0, then no current flows through C

$$V_G = \frac{100K}{600K} 12V = 2V$$

Guess Saturation Region for MOSFET

$$V_{GS} > V_{TH} \qquad V_{DS} > V_{GS} - V_{TH}$$

$$I_D = \mu C_{OX} \frac{W}{2L} \left(V_{GS} - V_{TH}\right)^2$$

$$I_D = 10^{-4} \frac{45.6}{2} \left(2-1\right)^2 = 2.28mA$$

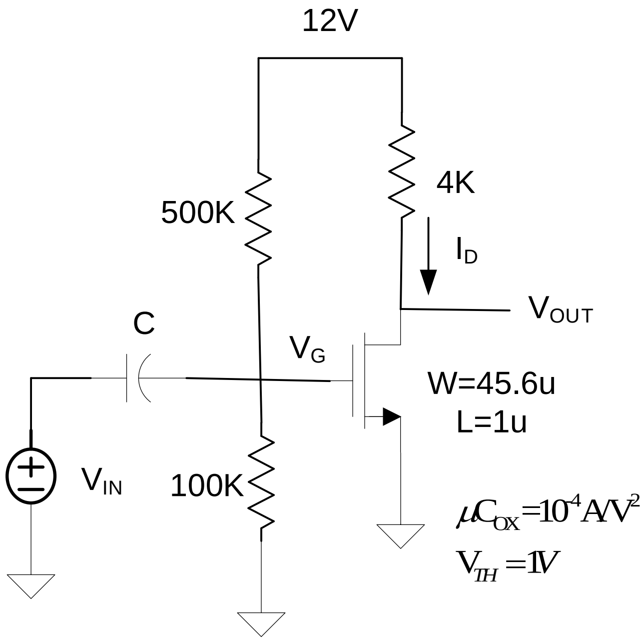
V_{OUT}=2.88V

Verify saturation $\quad 2V > 1V \qquad 2.88V > 2V - 1V$

Note: This circuit has the same current and same V_{OUT} as the previous circuit

Note: solution dependent upon W,L,V_{TH}, and uC_{ox}

Example: Determine I_D and $V_{OUT.}$ Assume C is large and V_{IN} is very small.



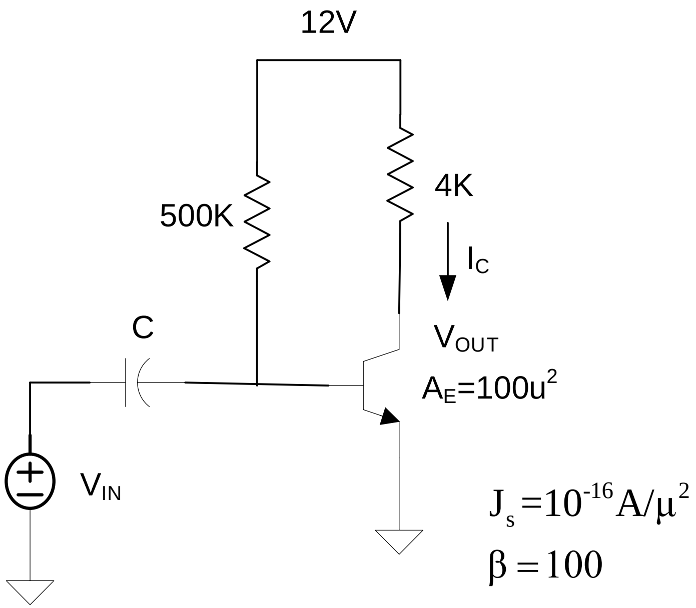
Solution:

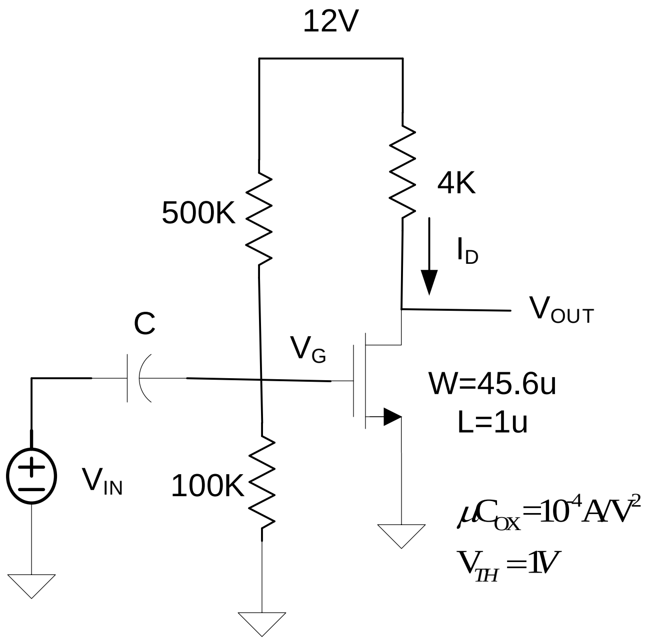
Assume V_{IN}=0, then no current flows through C so circuit is identical to circuit of previous-previous example so

$$I_C = 2.28mA \qquad V_{OUT} = 2.88V$$

Note: If C is large and V_{IN} is small sinusoidal signal of sufficiently high frequency, the voltage across C will not change so V_{IN} is from an ac viewpoint coupled directly to base. In this case, the circuit will amplify V_{IN} and the gain will be large due to the square-law relationship between I_D and V_{GS}.

Comparison





$I_C = I_D = 2.28mA$ $\quad V_{OUT} = 2.88V$

- Both circuits can serve as amplifiers
- Architectures very similar
- Will be shown later that the bipolar circuit has larger gain because exponential vs square law relationship

Stay Safe and Stay Healthy !

End of Lecture 20